

LOCATION OPTIMIZATION OF CONTINENTAL UNITED STATES STRIP ALERT SITES SUPPORTING HOMELAND DEFENSE

THESIS

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THESIS

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Abstract

With the dissolution of the Warsaw Pact and the fall of the Soviet Union, the number of alert aircraft dwindled to 14 aircraft located at 7 sites on September 11, 2001. After the terrorist attacks on the World Trade Center Towers and the Pentagon, the United States could not continue to endorse an outward looking air defense strategy. Terrorism completely changed the landscape of the air defense mission.

This research develops a location optimization model to optimally locate alert sites post-11 September to cover areas of interest in the CONUS. The model finds the minimum number of alert sites, minimum aggregate network distance, and minimized maximum distance given a range of aircraft launch times and speeds. The model is formulated as an Integer Program, and Microsoft Excel's[®] Solver™ Add-In is used to run the model. Finally, the optimal network configuration is examined by changing mixes of candidate alert sites to examine possible what-if scenarios. Sensitivity analysis is used to explore how much the optimal solution(s) change given fluctuations in input values.

This research provides air defense planners a tool to use in formulating an optimal strip alert network. By finding the minimum number of sites and the minimum aggregate distance to cover all areas of interest, duplication of coverage effort, dispersion of resources, and network response time is minimized. The results presented in this research should lead to a more efficient and effective air defense strip alert network to support homeland defense of the United States.

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LOCATION OPTIMIZATION OF CONTINENTAL UNITED STATES STRIP ALERT SITES SUPPORTING HOMELAND DEFENSE

I. Introduction

Background

For over two hundred years the United States has relied upon geographic positioning to defend itself from enemies. Allies in the North and South, coupled with expansive oceans in the East and West, insulated the United States from potential aggressors and mitigated the need for active homeland defense measures. The end of the Cold War and the subsequent dissolution of the Soviet Union signified the disappearance of the one enemy who provided a credible threat to defense of the American homeland. Not only did the collapse of the Soviet Union eliminate a direct threat to the United States, it also represented a change in the type of threats to the United States from conventional to unconventional adversaries. The United States no longer needed the massive stockpiles of weapons, armies, and anti-missile defense systems to deter the Soviet Army. Consequently, the homeland defense strategy of deterrence gave way to a leaner, flexible military force dedicated to combating unconventional threats such as terrorists and rogue nations.

Since the end of the Cold War, no unconventional threat to the United States has received more attention than terrorism. The tactics of terrorists are unbounded by the

traditional rules of warfare (White House, 2002:11). Terrorists employ a wide variety of tactics transforming objects of daily life into weapons that can inflict destruction on unsuspecting populations. Until the 1990s, terrorist attacks against American citizens primarily occurred abroad. However, the terrorist attacks on the World Trade Center in 1993 and the Alfred P. Murrah Federal Building in 1995 forced the executive branch of the government to seriously contemplate the effectiveness of the homeland defense policy in the United States. Both cases demonstrated the need for effective intelligence and greater attention to consequence management (Pollard, 2003:2). In 1998, President Clinton announced his approval of two important Presidential Decision Directives (PDDs). "PDD-62 and PDD-63 addressed counterterrorism and critical infrastructure protection respectively, and were the result of a series of related presidential and congressional initiatives" (Pollard, 2003:3). Although the directives increased recognition of the need for a coordinated approach to homeland defense, the directives did not prepare the country for the events of 11 September 2001.

The simultaneous terrorist attacks on the World Trade Center Towers and the Pentagon on 11 September 2001 brought homeland defense to the forefront of domestic policy. President Bush established the Office of Homeland Security by Executive Order on 8 October 2001 to coordinate the executive branch's efforts..."to detect, prepare for, prevent, protect against, respond to, and recover from terrorist attacks within the United States" (Bush, 2001). An important tool in the accomplishment of each of the aforementioned objectives remains the North American Aerospace Defense Command (NORAD) and its combat air patrol (CAP)/alert program. The joint United States/Canada command is responsible for protecting the skies over both nations;

however, the mission's volume and scope underwent considerable changes after 11 September 2001.

Before the attacks on New York and Washington, NORAD maintained 14 fighters on alert in the United States. That number was increased to over 100 aircraft on September 11. By the next morning, more than twice that many aircraft were placed on alert (Scott, 2002:32). "Since the events of September 11, more than 29,000 CAP sorties have been flown with more than 1,000 intercepts" (Hughes, 2003:35). Furthermore, the events of September 11 drove defense leaders to institute Operation NOBLE EAGLE, where the Air National Guard, Air Force Reserves, and active duty Air Force flew 24hour, fully armed CAP patrols over strategic areas of interest within the United States and placed scores of jets on alert status around the country. Although the initial response was extraordinary, it soon became evident to senior leaders that the 24-hour CAPs would have a significant impact on personnel and airframes if sustained for extended periods of time (Orletsky et al., 2003:4). Also, the manpower and equipment issues were exacerbated by the demands placed on the Air Force by Operations SOUTHERN WATCH, NORTHERN WATCH, and ENDURING FREEDOM. Subsequently, the bulk of the 24hour CAP patrols were scaled back in favor of a larger strip alert posture. Strip alert is the pre-positioning of air defense assets at predetermined alert sites or runways to respond to expected threats.

Presently, strip alert support is increased or decreased by NORAD depending on the threat level disseminated by the Department of Homeland Security. Intelligence information dictates the places within the United States that receive the most attention of CAP support. Air Combat Command (ACC) serves as the primary force provider for the

air defense mission. Although the alert aircraft numbers and sorties fluctuate with the threat level and intelligence information, the requirement to base aircraft and crews as well as cover certain areas of interest in and around the United States remains relatively constant. With alert requirements ongoing for the foreseeable future in the war against terrorism and to provide homeland security, NORAD must administer the steady-state strip alert program in the most efficient and effective way possible to maximize their given assets.

This research seeks to develop a mathematical model to optimize the coverage area of the strip alert locations, while minimizing both travel time of aircraft and the number of alert locations. This research is intended to assist the Department of Homeland Security, NORAD, and ACC to improve the effectiveness and efficiency of the air defense network, while bolstering homeland defense capabilities of the United States. Additionally, this research will conduct sensitivity analysis on results of the model to show how changes in model parameters affect the overall system and to evaluate alternative scenarios.

Problem Statement

Prior to September 11, NORAD kept 14 fighter aircraft on alert status at seven geographically selected alert pads/bases around the United States. Since the assets were not utilized at the level of post September 11, the location of the alert facilities were determined by existing infrastructure, with less emphasis on efficient location for site coverage. With the establishment of the National Strategy of Homeland Security, alert aircraft will play a dominant role in each of the following strategic objectives: "1.

prevent terrorist attacks within the United States; 2. reduce America's vulnerability to terrorism; and 3. minimize the damage and recover from attacks that do occur" (White House, 2002:vii). Given the greater reliance on CAP aircraft, the increase in aircraft on alert status, and the increase of desired aircraft coverage area post-September 11, CAP assets need to be placed at optimum locations supporting the objectives of NORAD and the National Strategy of Homeland Security in order to efficiently utilize these assets. This seeks to bring to bear the right number of aircraft on the enemy at the right place, in the right time frame, and in the most effective and efficient manner possible to guarantee defense of critical assets in the United States.

Research Questions

What are the optimal strip alert locations in the Continental United States for aircraft in support of homeland defense of the United States? In order to build an effective model and answer the overarching research question the following investigative questions will be explored:

- 1. What is the history of the alert network (Cold War to present)?
- 2. What are the alert system objectives and their relative importance in the overall air defense network?
- 3. What is the best method for solving the strip alert network problem and what are the critical model parameters leading to a specific modeling method?
- 4. How do the optimal solutions change when adjustments are made to critical model parameters?

Research Methodology

The methodology used in this research will consist of a four-stage process. First, an investigation of the objectives of the strip alert network and the critical model parameters will be performed to aid in the location modeling technique selection process. Second, a study of location modeling techniques will be conducted to determine the most appropriate technique(s) to use in construction of the model. Once the modeling technique is selected, the model will be built utilizing the critical parameters identified in stage one. Third, the model will be run to determine the optimal strip alert locations given the objectives and parameters. Finally, adjustments will be made to key model parameters and the model re-run to evaluate the sensitivity of the optimal strip alert network solution obtained in step three. Options and recommendations will be presented given different scenarios generated by adjusting the model parameters.

Data

The data required for this research will be provided by the ACC Office of Homeland Security and the First Air Force Air Operations Center. This data will include the required runway length and airfield requirements of the overall network. Data on the operating characteristics of the aircraft utilized in the network including historical launch times by base will be provided. The data will contain the desired number of aircraft and ground spares to be placed at each site. Also, the data will show the areas of interest to be protected as well as the desired response time to each area of interest. Each candidate airfield and area of interest will be identified by its distinctive latitude-longitude coordinate. The latitude-longitude coordinates will be obtained from First Air Force

personnel and from the Department of Defense (DoD) Flight Information Publication (Enroute).

Assumptions

This research assumes that Air Force F-16 and F-15 aircraft will be utilized in the network and that aircraft launch times as well as aircraft flight speed will follow historical trends. Also, this study assumes that the desired number of aircraft will launch and arrive successfully when scrambled and that the F-16 and F-15 aircraft are perfect substitutes throughout the network. Also, this research assumes that the number of aircraft placed on alert at any site has no bearing on response time. Airborne tanker aircraft, maintenance, and ground equipment support are assumed to be available when needed. To produce a greater number of possible options, this model assumes that any candidate airfield possesses an equal probability of selection as any other candidate and that the overall network contains no airspace restrictions. Finally, this research assumes that the network is not constrained by the number of alert sites that can be selected.

Scope and Limitations

The goal of this research is to provide an optimal air defense alert network for the Continental United States given the objectives of minimizing aircraft response times and the number of alert locations, while covering all demand areas within a given distance constraint. Although the Navy and Marines possess fixed wing aircraft, only Air Force F-16 and F-15 aircraft are considered in the evaluation. This research will not assess the probability of the optimal network successfully defending a protected area given an

attack or act of aggression. The study does not explore the infrastructure costs associated with conversion to the optimal network nor does it use costs, lack of facilities, or support as a constraining factor in site selection. No consideration is given to the temporal variation or spatial distribution of demand of the overall network. Finally, this research does not take into account political objectives in evaluating the desirability of a particular strip alert site candidate.

Summary

This chapter provides the justification for building a mathematical model to optimize the location of CAP strip alert sites. Optimizing the location of these critical sites will allow defense planners to maximize performance of the overall alert network while minimizing the amount of resources required to cover identified areas of interest. Ultimately, this research will provide defense planners with a tool capable of generating optimal network configurations given changes in critical model parameters.

Chapter II discusses the evolution of the air defense alert network from the Cold War to present, including the impact of the world political climate on overall network configuration. Additionally, the chapter covers the changes in homeland defense policy. The chapter examines previous research conducted in the area of location analysis. Furthermore, this chapter presents and reviews a number of different location analysis techniques and tools, providing the pros and cons of each tool's suitability to the overall objectives of the network.

Chapter III presents the methodology of this thesis. It covers the collection of the data, the selection and formulation of the location modeling techniques and the specific model set formulations to be analyzed in Chapter IV.

Chapter IV discusses the results of each different model set. It describes the sensitivity of the models given changes to some of the critical model parameters. The chapter also discusses possible "what if" scenarios that could be posed to the model.

Chapter V talks about the conclusions and inferences that can be drawn from the model results as well as discussion of the model's use to the Department of Homeland Security, ACC, NORAD, and the Air Force Base Realignment and Closure (BRAC) Office. Also, the model's limitations are discussed. Finally, recommendations are provided as to how to extend the use of this model and to highlight future research possibilities.

II. Literature Review

Introduction

This chapter presents an overview of the evolution of the fighter alert network from the Cold War to present and of the literature related to location analysis problems. The chapter covers common methodologies used to solve location analysis problems to include previous research in the field of location analysis. These modeling techniques include: maximum distance models, p-dispersion models, and total average distance models. Techniques utilized hinge directly on the taxonomy of the location problem. Common taxonomies of location analysis problems are briefly discussed. Furthermore, the chapter explores the pros and cons of decision analysis tools including optimization techniques and heuristics that can be employed to solve location analysis problems. Finally, an overview of complexity analysis is presented to aid in selection of a decision analysis tool and location modeling technique(s).

Increasing the efficiency and effectiveness of the CONUS fighter alert network is an important objective of the United States Air Force (USAF), the DoD, and the Department of Homeland Security. With increasing demand being placed on fighter airframes around the world due to numerous peacekeeping and contingency operations, placing CONUS strip alert assets at the optimum locations remains paramount. Correct placement will help ensure that damage caused by hostile attack to critical areas and infrastructure around the United States can be mitigated or avoided with the appropriate amount of force.

History of the Alert Network (Cold War to Present)

In 1948, at the outset of the Cold War, the United States embraced a homeland defense policy aimed at preventing any potential Soviet attack on the United States. These potential attacks were believed to happen via long range bomber or later by intercontinental ballistic missile (ICBM). The fighter interceptor alert posture was geared toward stopping the Soviet-manned bombers from delivering their payloads on American soil. The U. S. fighter aircraft forces were organized by interceptor squadron. Tactical Air Command (TAC), Strategic Air Command (SAC), and the Air National Guard (ANG) contributed fighter and radar forces to the air defense mission of Air Defense Command (ADC). The ANG comprised the major source of the wartime air defense units; however, the ANG lacked the necessary air defense training, and. many of the units were transitioning back from World War II. To further complicate matters, any forces belonging to TAC and SAC who possessed an air defense mission would serve ADC as a secondary duty.

The early interceptor operations were crude at best.

For example, on 27 March 1948, Air Force Chief of Staff, Carl A. Spaatz directed SAC to move the 27th Fighter Group from Kearney, Nebraska, to McChord AFB, Washington, to protect the Atomic Energy Commission's plant at Hanford. The P-51 aircraft provided by SAC were useless in the bad weather experienced in Washington. Also, SAC aircrews were not trained properly in interception techniques and the technicians assigned to ground radars had not mastered the art of directing an interceptor to a precise point in the air. (McMullen, 1973:24)

In addition, the fighter interceptor force was effective only during daylight hours.

Although the early air defense mission of the United States had been performed with limited success, ADC was directed to expand the Northwest operations to the Northeast

and the Southwest on 23 April 1948. This expansion brought the air defense fighter force to seven squadrons at four bases (McMullen, 1973:28). However, this expansion was soon to be trumped by another reorganization. Problems with mission delineation, organizational boundaries, and escalation of threats got the immediate attention of DoD and USAF leaders.

On 1 December 1948, the Air Force created Continental Air Command (ConAC) (ADC, 1962:20). ConAC was centered on the air defense mission, and inherited TAC and ADC as operational commands. The ConAC realignment created a double-duty fighter force. ADC and TAC squadrons with a primary air defense mission were given a secondary mission of ground support. Units with a primary ground support mission were expected to fulfill air defense as a secondary objective. As a result of the ConAC reorganization SAC was tasked to provide nine squadrons to the air defense mission. "By the stroke of the pen, the air defense fighter force increased from seven squadrons on four bases to 16 squadrons on six bases" (McMullen, 1973:28). This level was considered effective until 29 September 1949, when President Truman announced publicly that the Soviet Union had exploded an atomic device in August (McMullen, 1973:32). The announcement heightened U. S. worries and caused a quickening of CONUS air defense preparations.

The atomic explosion in the Soviet Union caused the United States and Canada to closely examine the North American defense network. On 1 June 1950, the first Canadian-U. S. Emergency Air Defense Plan was approved (ADC, 1962:23). Around-the-clock alert fighter interceptor alert operations began on 27 June 1950, and President Truman authorized intercept and engagement of aircraft anywhere in the U. S. on 24

August 1950 (ADC, 1962:24). Subsequently, the USAF approved a wider interceptor force dispersal plan. On 17 July 1950, the 20 active duty interceptor squadrons were dispersed from seven locations to fourteen (ADC, 1962:22). However, the locations were predominately in the Northwest, Northeast, and Southwest. Although the new dispersal plan put the active duty squadrons at a greater number of locations, the plan did not outline the incorporation of ANG forces in case of contingency.

At this point in time, the postwar reorganization of the ANG was about two-thirds complete; however, the enormous problem of training ANG forces in air defense tactics remained. In an emergency, the USAF estimated that approximately 70 percent of the total interceptor force would be provided by the ANG (McMullen, 1973:34). To make matters worse, ANG forces were under control of the individual states during peacetime, exacerbating ConAC's problems of fitting the ANG units into the overall air defense mission. "It was recommended, instead, that ANG units with an air defense mission be given an air transport or ground support mission and the void in air defense be filled with regular air defense squadrons" (McMullen, 1973:34). The respective state governments opposed this solution because the ANG was proud of its important role in the air defense mission and did not want to be given air transport duties. Therefore, for the first time in history, the state governors submitted to a greater degree of ConAC control to improve the readiness of ANG units with an air defense mission.

Although the states agreed to allow the air defense training of their personnel, ConAC encountered problems in the timely activation of the ANG forces. This was about to change. In November of 1950, the USAF decided to stand down ConAC and recreate an independent ADC effective 1 January 1951, based at Colorado Springs,

Colorado (McMullen, 1973:38). ADC inherited 23 fighter squadrons from ConAC and 38 dedicated ANG fighter squadrons that were assigned an air defense mission, bringing the pool of ADC fighter interceptor squadrons to 61 (ADC Historical Services, 1954:24). ADC attacked the ANG mobilization problem by petitioning the USAF for federalization of selected ANG units.

In the first three months of 1951, the USAF, through Presidential approval, brought 21 of the 38 ANG fighter squadrons to active duty under the ADC umbrella for a period not to exceed twenty-one months. This increased the number of active duty fighter interceptor units to 44. (McMullen, 1973:44)

Although the federalization of the ANG forces filled holes in the alert network, many of the ANG units were not optimally positioned and still lacked the necessary training.

Soon after the ANG forces were federalized, ADC Headquarters changed the permanent location of 10 units to more advantageous air defense positions. ADC decided to allow units to locate in different regions because current positioning would have been a waste of resources due to the fact that many of the fighter units were located in the same areas. "Effective use required allocation on a basis of priority of targets, forces available for defense of these targets, and the capability of these forces (ADC Historical Services, 1954:38). Most new basing positions for ADC interceptor squadrons were co-located with active early warning radar systems. The location changes are shown in Table 2.1.

Table 2.1 Early Defense Location Changes (Early 1951) (McMullen, 1973:44-45)

FROM	TO
Reading (Pennsylvania)	Dover Air Force Base (AFB) (Delaware)
Stout Field (Indiana)	Scott AFB (Illinois)
Kellogg Field (Michigan)	Selfridge (Michigan
Mitchell Field (Wisconsin)	Truax Field (Wisconsin)
Bradley Field (Connecticut)	Suffolk County AFB (New York)
Holman Field (Minnesota)	Wold-Chamberlain (Minnesota)
Kirtland AFB (New Mexico)	Long Beach Airport (California)
Sioux Falls Airport (South Dakota)	Ellsworth AFB (South Dakota)
Baer Field (Indiana)	Sioux City Airport (Iowa)
Berry Field (Tennessee)	McGhee-Tyson Field (Tennessee)

Once the ANG units were federalized and the necessary moves were conducted, ADC initiated a vigorous training program in to orient rusty ANG pilots and ground crews to the air defense mission. Although the new ADC was able to shore up some of the previous network deficiencies, a realization existed throughout the DoD and Congress that the new system was merely a copy of the World War II system with better equipment.

Senior DoD and U. S. leaders knew that great strides had been made in the CONUS air defense fighter and radar network; however, much work was left to be done. "The most optimistic estimates of 1951 were that the air defense establishment might destroy 30 percent of an invading bomber force" (McMullen, 1973:49). Subsequently, the USAF commissioned the Massachusetts Institute of Technology (MIT) in March 1951 to find ways to improve the air defense network. The analysis effort became known by the code name PROJECT CHARLES. In the PROJECT CHARLES report, MIT scholars theorized that the advancement of the digital computer would have a dramatic impact on the speed of threat identification and data transmission within the air defense network. Analysis showed that it currently required an average of 8.1 minutes to pass an

enemy aircraft sighting from the observation post to the alert interceptor aircraft (McMullen, 1973:48). PROJECT CHARLES scientists believed that the technological advances would dramatically reduce the transmission time to the point that the air defense network could feasibly experience a 60 to 70 percent successful intercept rate (McMullen, 1973:54). Ultimately, the report generated by the scientists caused the commissioning of the Sumner Study Group who deemed that the existing plan and state of the current air defense system was inadequate in the summer of 1952.

DoD leaders acted on the Sumner Study Group's assessment immediately.

On 26 August 1953, Admiral Arthur C. Radford, in his first press conference as Chairman of the Joint Chiefs of Staff (JCS), said that the Soviet possession of the hydrogen bomb made it imperative that the United States improve its air defenses. (McMullen, 1973:63)

Admiral Radford pushed for an interceptor expansion goal of 69 squadrons by 1955. Through the support of leaders like Admiral Radford, ADC achieved the expansion authorization of 69 squadrons in December 1953 (ADC, 1962:33). Radford believed that it was imperative to improve the quality of the overall fighter force, thus, the conventional (propeller-driven) aircraft disappeared first, with the last of the day jets dropped in early 1955. Afterward, all fighter interceptors were all-weather jets. ADC also increased the number of permanent radar stations in the U. S. to 90 (McMullen, 1973:64). ADC did not let the improvements end with more forces and better technology.

In the mid 1950s, the command began experimenting with a two jet, five-minute launch daytime alert posture at Syracuse, New York, and Hayward, California, to test the feasibility of standing ANG forces on rapid response (ADC, 1962:33). The success

realized during this trial alert period was a harbinger for the posture on 1 October 1954, when ADC put 17 ANG squadrons on dawn-to-dusk alert (ADC, 1962:39). While ADC bolstered the alert posture and force numbers through increased funding and better training, lawmakers and defense officials planned and debated the future of the U. S. air defense network.

Although the DoD and the Eisenhower administration began spending more money on air defense in the mid 1950s, many powerful democratic senators felt that the actions were too little too late. While Congress debated funding and the general direction of the air defense mission, the JCS directed establishment of Continental Air Defense Command (CONAD), effective 1 September 1954, with headquarters at Colorado Springs (McMullen, 1973:68). Since CONAD was a JCS command, all three services contributed forces. Specifically, the Army supplied antiaircraft weapons, the Navy contributed picket ships and limited numbers of aircraft, and the Air Force provided aircraft, radar, and most of the CONAD staff (McMullen, 1973:69). ADC still existed, but under the CONAD umbrella. Congress continued to debate potential funding levels. The report issued by the Sumner Study Group called for an air defense budget of more than a billion dollars a year. When the magnitude of the funds involved became apparent to lawmakers, it was obvious that the funds available for air defense purposes would fall short of projected costs.

Congress's decision to fund at a reduced level caused USAF and DoD leaders to embrace a strategy of selective upgrade. Much of the funding in the DoD budgets of the late 1950s went to improve the early warning radar systems in the air defense net. The desired expansion of the interceptor force was a victim of the radar upgrade priority.

In September 1956, ADC was informed that in a 1957 Air Force of 137 Wings it would be permitted 80 interceptor squadrons. In October of 1956, however, it learned informally from USAF that because of the fund shortage it would be limited to a total of 68 squadrons. In the absence of formal instructions, however, ADC activated its 69th squadron in November 1956. (McMullen, 1973:75).

The interceptor squadron level of 69 coincided to the level approved in December 1953. Air defense upgrades came to a halt at the close of 1956, and every element of the manual air defense system took a financial beating in 1957. The manned interceptor force reached its apogee of 69 manned and equipped squadrons in the middle of 1957 and began a decline which continued to the end of the Cold War (McMullen, 1973:83). Although funding for the air defense mission declined significantly in 1957, Canada and the United States took a large step toward the integration of North American air defense forces.

On 12 September 1957, NORAD was established and headquartered at Colorado Springs (ADC, 1962:49). The joint U. S.-Canadian command integrated all North American air defense forces under a single command, and, the primary mission of NORAD was to intercept any Soviet long-range bombers attacking over the North Pole (General Accounting Office (GAO), 1994:14). CONAD remained, but NORAD became an umbrella for CONAD while adding the Canadian Forces Air Defence Command. When NORAD was established, the fighter interceptor force was at its maximum.

At the height of the Cold War, when the threat from the Soviet Union's long-range bombers posed a major strategic threat, Aerospace Defense Command maintained 1,500 interceptor aircraft at more than 100 air defense "alert sites" around the nation. Fighters stood cocked and ready, 24 hours a day to scramble and repel an attack. (Kitfield, 2002:62)

The alert sites were spread throughout the CONUS, with the bulk of them around the borders, and the posture was outward looking in anticipation of the Soviet threat. This outward looking strategy became a major attribute of U. S. air defense throughout the Cold War. However, the fighter interceptor strength would soon begin to decline. By 1960, the fighter interceptor force strength was reduced by approximately 300 jets to a strength of 1,200. This was accomplished through numerous fighter interceptor squadron deactivations conducted in the latter 1950s. The deactivations were a reaction to reduced funding as well as to a change in Soviet strategy.

In 1957, the Soviets successfully launched their first ICBM. By the early 1960s it was apparent that the Soviet Union was putting more emphasis on its ICBM program and less on its manned bomber network.

In response, the United States built a space-based surveillance and missile-warning system to detect and track airborne threats worldwide. NORAD was given responsibility for this system, thereby adding to its mission the tactical assessment and warning of a possible air, missile, or space attack on North America. (GAO, 1994:14)

The importance of the air interceptor mission dwindled to the point that NORAD authorized ADC commanders to scramble one aircraft, instead of two, to perform identification intercepts of unknown aircraft in CONUS airspace on 12 January 1960 (ADC, 1962:59). Although attention dwindled, defense leaders continued to modernize the interceptor fleet. In early 1961, the regular fighter-interceptor force completed its move to century series all-weather aircraft, including the F-101B, F-102A, and F-106B (ADC, 1962:63). Also, the ANG began to take a more important and active role in the day-to-day air defense of the U. S by placing greater numbers of aircraft on alert in the air defense network.

The ANG interceptor squadrons remained under state control; however, by late 1962, the units rotated alert duty between 16 of the 22 squadrons who were obligated to air defense. Units were tasked with a minimum of dawn to dusk alert. The remaining six squadrons were permanently committed to a continuous, around-the-clock, seven-days-a-week alert (ADC, 1962:110). ADC also made other changes to the fighter interceptor alert posture. ADC changed the requirement from two aircraft on five-minute alert to one third of an interceptor squadrons' aircraft on 15-minute alert status (McMullen, 1973:125). Also, in 1963, ADC outlined a different interceptor dispersal plan. "The plan called for deployment of half the aircraft of most interceptor squadrons to a predetermined dispersed operating base upon receipt of warning of an ICBM attack" (McMullen, 1973:125). This plan was immediately put to the test.

In October 1962, the United States and the Soviet Union engaged in a confrontation over the Soviet Union's installation of ballistic missiles in Cuba. "At that time ADC promptly dispersed 161 interceptors from 28 squadrons to 16 dispersal bases" (McMullen, 1973:125). The United States found that the dispersal bases were not ready in terms of infrastructure or supplies, but the action proved that the majority of the interceptor forces could be moved into strategic locations or out of harm's way on short notice (McMullen, 1973:126). Secretary of Defense McNamara endorsed a permanent dispersal plan, where four to six aircraft of each dispersing squadron would be moved to their away-from-home location. The practice of deploying fighters to strategic CONUS locations continues today. Although the DoD made operational changes to the configuration of the fighter defense network throughout the 1960s, the numbers of aircraft assigned to the air defense mission continued to decline.

The manned air defense capability of ADC and NORAD was systematically reduced throughout the mid and late 1960s due to the decline of the Soviet manned bomber threat. This trend was quickly reversed in 1972 and 1973, because a plane-load of Cuban officials went undetected through the Southern United States until it requested landing instructions from the airport tower in New Orleans on 26 October 1971 (McMullen, 1973:221). A Congressional investigation later revealed that no significant air defense existed along the 1,500 mile southern border from California to Florida (McMullen, 1973:221). In May 1972, Secretary of Defense Melvin R. Laird established Southern Air Defense (SAD). "SAD stationed alert interceptors at four locations—Tyndall AFB, Florida, Ellington AFB, Texas, New Orleans, Louisiana, and Tucson, Arizona" (McMullen, 1973:222). This action was a brief respite to the drawdown of the total fighter interceptor forces.

Due to the waning threat of the manned Soviet bomber, the United States continued its drawdown of fighter interceptor forces in the 1970s. Funding and Congressional support for the air defense mission decayed, and by the late 1970s only 340 manned interceptors remained in the network (Burda, 1986:3). Ultimately, the drawdown of the interceptor forces led to the deactivation of ADC on March 31, 1980. ADC's air defense assets were transferred to TAC, and its space and missile warning assets were transferred to SAC (Ingelido, 1988:6). NORAD continued exercising operational control over these forces; however, the forces were owned by TAC and SAC much in the same way as they were at the beginning of the Cold War. The objectives of NORAD's new manual or fighter air defense network were threefold: First, provide warning of air attack to the National Command Authority; Second, to prevent Soviet

bombers from entering America's heartland; Third, serve as the gatekeeper for North American airspace (Committee on Armed Services, 1981:3). Even though the forces were excused from exclusive ADC control, the drawdown of fighter alert numbers continued through the 1980s and 1990s.

The reduction in fighter interceptor forces hit overdrive in 1991. In December 1991, the Soviet Union and the Warsaw Pact were dissolved. This dramatically changed the threat landscape upon which NORAD based its operations for over thirty years.

NORAD recognized this drastic reduction in the military threat and determined that sufficient warning time existed to reconstitute forces needed to meet reemerging threat of the magnitude of the former Soviet Union. Consequently, NORAD revised the justification for its core forces, emphasizing peacetime air sovereignty. (GAO, 1994:15)

Air sovereignty involves the control of the territorial airspace of North America. This was a departure from the air defense mission. Subsequently, the focus of NORAD moved from defending the U. S. from the Soviet Union to drug interdiction with the fall of the Warsaw Pact. This led to further cuts in fighter interceptor alert numbers. In 1994, NORAD reduced the number of alert sites to 14 and the number of alert aircraft to 28 for peacetime air sovereignty (GAO, 1994:16). The pre-1994 network is illustrated in Table 2.2

Table 2.2 Air Defense Units and Alert Sites, 1989-1992 (GAO, 1994:17)

Air defense unit/alert site	Status ^a
Atlantic City, New Jersey	1
Burlington, Vermont/	1
Langley AFB, Virginia	3
Duluth, Minnesota	5
Tyndall AFB, Florida	3
Ellington AFB, Texas/	1
Holloman AFB, New Mexico	3
Fargo, North Dakota/	5
Kingsley AFB, Oregon	3
Fresno, California/	1
Castle AFB, California	4
George AFB, California	4
March AFB, California	3
Great Falls, Montana/	4
Davis-Monthan AFB, Arizona	3
Jacksonville, Florida/	1, 4
Homestead AFB, Florida	4
Key West, Florida	3
Niagara Falls, New York/	5, 6
Charleston, South Carolina	4
Otis, Massachusetts/	1
Bangor, Maine	3
Loring AFB, Maine	4
New Orleans, Louisiana	2
Portland, Oregon/	1
McChord AFB, Washington	4
Selfridge, Michigan/	5, 6
Seymour Johnson AFB, NC	3
Elmendorf AFB, Alaska	2

^a1, Dedicated air defense unit with home station alert site; 2, dual-tasked unit; 3, detached alert site; 4, alert site closed or planned to close; 5, no home alert; 6, changing missions

The composition of the network in 1994 contained primarily ANG forces. In its report, *GAO/NSIAD-94-76*, the GAO recommended in concert with the Chairman, Joint Chiefs of Staff, to use general purpose and training squadrons instead of the ANG to perform the air defense mission (GAO, 1994:9). This action, which was opposed by the DoD and USAF, was estimated to save approximately \$370 million annually.

The proposed plan to divorce the ANG from the air defense mission did not reach implementation. As state militia, ANG forces work closely with the governments of their respective states and ANG leaders are entrenched in the political networks. "While under state authority, Guard forces are not restrained (as are active duty forces) by the posse comitatus law forbidding the military from performing domestic law enforcement functions" (Kitfield, 2002:63). This makes the Guard a natural fit for the air defense mission. Keeping the Guard as the primary force provider for the air defense mission was not the only thing that the DoD and USAF decided was best. The complete demise of the Soviet Union forced defense leaders to shave alert site and aircraft numbers even further after the 1994 cuts.

From the end of 1994 to 2001, the DoD, NORAD, and the USAF cut the number of active alert sites to seven and the number of alert airplanes to 14 (Hebert, 2002:50; Scott, 2002:32). This was the fiscally responsible decision given that the one credible threat to the U. S.—the Soviet Union, was gone. Furthermore, the locations that the DoD retained were strictly around the periphery of the United States looking for incoming danger just as the 1,500 aircraft had done at the height of the Cold War (Hebert, 2002:50; Kitfield, 2002:62). Also, much like the Cold War days, the F-15 and F-16 fighters sitting alert maintained a 15 minute response time established years before to counter a foreign threat (Hebert, 2002:52). However, on 11 September 2001, the terrorist attacks on the World Trade Center Towers in New York City and the Pentagon in Washington D. C. did not occur from outside the U. S., but from within its borders. In response, NORAD frantically tried to scramble the alert fighters to intercept the hijacked airplanes, but the aircraft could not reach the terrorist hijackers in time to stop them. "It was an F-15 unit

from the Massachusetts ANG that scrambled to New York and similarly, an ANG unit stationed at Langley AFB, Virginia, raced to the Pentagon" (Hebert, 2002:48).

Unfortunately, the events of 11 September illustrated how deep the post-Cold War alert force had been pared under actual operational conditions. As discussed in Chapter 1, terrorism completely changed the dynamics of air defense. The air defense network could no longer operate under an outward looking defense strategy. The threat as well as the air defense network needed to be re-evaluated to locate and build a future air defense posture.

It quickly became time for NORAD to change the number of alert jets and alert sites as well as adopt an inward and outward looking air defense philosophy. Within hours of the hijackings, NORAD launched enough airplanes to perform continuous CAPs over 30 locations around the United States and stood scores of airplanes on full strip alert (Hebert, 2002:52). As of February 2002, the USAF and NORAD had increased the number of alert bases to 26, with four fighters ready to go at each site (Hebert, 2002:52; Mann, 2002:26). The increased number of 24-hour CAPs continued until well into 2002, when it became evident that personnel availability and airframe serviceability would be severely impacted if the pace continued (Orletsky et al., 2003:4). On 23 February 2002, in an Associated Press interview at Tinker Air Force Base, Air Force Secretary, Mr. James Roche, indicated that he would prefer an adjustment that would place Air Force fighter jets on strip alert at certain bases around the country as opposed to continuous CAPs (Constant Air Patrols, 2002). Moving to such a posture has received a great deal of recent attention from senior Air Force leadership, due to the effect on mission effectiveness.

The current air defense network fluctuates in spatial and temporal nature. The number of jets placed on alert, as well as the number and location of CAPs, varies with the threat level disseminated by the Department of Homeland of Security and intelligence information collected from many different sources such as the Federal Bureau of Investigation and the Central Intelligence Agency. NORAD alert postures in terms of levels, numbers of aircraft, supporting forces, and CAP coverage are classified and can be obtained through official channels with the proper security clearance and need to know. Also, the current alert sites are considered sensitive information and are not divulged in this thesis. However, a list of current alert sites can be obtained from NORAD, the First Air Force Air Operations Center, or the ACC Office of Homeland Security with the proper credentials. Although NORAD currently exercises a tiered air defense response system, the requirement to constantly maintain aircraft on alert remains an important part of U. S. air defense. Thus, it is imperative from an operational standpoint that NORAD place aircraft at optimum locations around the CONUS to promote overall network effectiveness and efficiency.

Evolution of Location Analysis

Location, location, and location. These words have been uttered since the infancy of man's drive to optimally locate supply centers responding to required demand. For example, public services such as fire departments and ambulatory services must be located close enough to demand centers in order to provide timely service. Failure to do so could result in damage to property or even death. Additionally, private enterprises must also be optimally located. Locating a shopping mall too far away from customers

can adversely affect business solvency and profitability. To this end, for more than 120 years, mathematicians, analysts, operations researchers, and management science scholars have tried to devise algorithms and techniques to identify optimal locations given a wide variety of problem parameters, resource constraints, and model objectives. Not only is location analysis useful to public and private commercial enterprises, but it is also relevant for use in the military.

From a public service and resource dispersion stand point, it is imperative that military bases be optimally located. To achieve the most efficient use of homeland defense resources, it is important that the Air Force locate strip alert sites at the most advantageous locations to minimize the number of resources required while maximizing performance of the overall strip alert network. To make these objectives a reality, this research intends to rely on location modeling. In order to choose the correct location modeling technique, a thorough study is conducted of location analysis problems, taxonomies, solution approaches, and applications.

Early Location Modeling

One of the early location analysis pioneers, Alfred Weber, attempted to find the most efficient point of production between raw material sources and required markets in order to build the most efficient overall network (Friedrich, 1957). Weber's system was based on utilizing geometric procedures called isodapanes to develop the most cost efficient network. An isodapane is the minimum total-transport cost point. Weber's method located production centers at the minimum total-transport cost point based on the process of the particular industry. For example, weight losing activities such as mining for gold would locate production facilities closer to the raw materials because of the

prohibitive transport costs of shipping waste. Conversely, weight gaining activities such as distributing soft drinks would locate production facilities closer to the markets because weight is added to the soda syrup at the last moment in the form of water, and water is ubiquity. Since the Weber Model was based on linear production relationships with no adjustment for economies of scale, as well as single objective in nature, it was limited to dealing with simple, single-site transport costing problems. Not only was the Weber Model one of the early location modeling algorithms, gravity models were introduced in this period as well.

In 1929, William J. Reilly developed a retail gravity model by applying the concept of spatial interaction. Reilly's gravity model is built on the premise that the interaction between two subareas is proportional to their activity levels, but inversely related to their spatial separation (Chan, 2001:17). Reilly's (1929) gravity model uses the number of business activities, people, and store sales as an index size and the basic measure of the attractiveness of a central place. The objective of the model is to find the point, based on the previous factors, where the consumer is indifferent between two different locations. This allows the calculation of an optimal trade area based on location. Hotelling (1929) also used the concept of spatial interaction to locate facilities based on pricing behavior of firms and consumer transportation costs. "In Hotelling's model, products differ only in one dimension, such as the stores that sell them" (Carlton and Perloff, 1999:216). Although these early gravity models were useful, they were limited in their application.

The Reilly and Hotelling models produced acceptable solutions when central places were easily distinguished; however, the models did not handle large population

gravity models were limited to solving simple market area, small number of location problems. The models were effective at evaluating interactions between small numbers of sites in rural areas, but overlapping markets and multiple locations in large population centers produced problems too complex for the models to effectively solve.

The shortcomings of the Weber Model and the early gravity models continued for the next 34 years because little progress was made in the area of location analysis. Many location theorists strayed from the challenge, because many believed that the complexities represented in such problems were impossible to solve analytically (Ghosh and Rushton, 1987:1). Therefore, theorists relied primarily on graphical approaches used in the Weber Model to effectively solve problems in simplified environments. The exclusive use of graphical methods continued until the early 1960s when several researchers (Kuhn and Kuenne, 1962; Cooper, 1963; Kuehn and Hamburger, 1963) developed mathematical algorithms that could be utilized with graphical methods to solve the general facility location problem (Ghosh and Rushton, 1987:2). With the addition of the algorithms, the Weber methodology was capable of solving problems in a complex environment as well as optimally locating multiple numbers of facilities.

The Classic Phase of Location Modeling

Locating multiple facilities presents the need to allocate demand to the respective locations. The decision of where to locate the facilities and where to allocate the demand simultaneously was the beginning of location-allocation modeling (Ghosh and Rushton, 1987). The Weber Model's shortcoming of locating multiple facilities was addressed by Cooper (1963). Cooper's research developed the classic facility location problem on a

plane, which minimizes costs for a multiple location network. Cooper's heuristic for minimizing shipping costs for multiple facility location was named the plane p-median problem.

P-median problems seek the location of p supply centers to minimize the demand weighted aggregate distance. The p-median problem was extended to solve for a network with discrete locations in Hakimi (1965) and ReVelle and Swain (1970). "The development of the network formulation of the p-median problem greatly extended the range of situations in which location-allocation models could be applied" (Ghosh and Rushton, 1987:2). Not only did the evolution of the p-median problem allow the application of location-allocation techniques to a greater number of circumstances, but it also drove the development of more efficient algorithms for solving location problems.

The three main heuristic algorithms developed after the p-median problem was the Greedy Algorithm, the Drop Algorithm, and the Interchange Algorithm. Kuehn and Hamburger (1963) developed the Greedy Algorithm to locate facilities incrementally by least cost until p facilities are located. The Drop Algorithm, developed by Feldman, Lehrer and Ray (1966), on the other hand, starts with facilities located at all possible sites and iteratively drops the facility at each stage with the least impact on the objective function (Ghosh and Rushton, 1987:3). The Interchange Algorithm, developed by Teitz and Bart (1968), is built around the selection of a set of p sites and an original minimum objective function value computed from the sites. Then, sites not in the set are substituted for each site in the set and the objective function value is recalculated each time. Substitution is continued until the value of the objective function is minimized. Not only did the evolution of the p-median problem give rise to the development of new

heuristics, but it also resulted in the greater application of mathematical programming methods such as linear programming.

In many instances, in order to solve the p-median problem using linear programming (LP), the integrality constraints must be dropped or relaxed. ReVelle and Swain (1970) found that the solution to the relaxed problem is often all-integer and therefore exact. This condition usually occurs when there is no fractional demand, when location allocation must occur at one or more of the nodes of a network, and when two distinct sets of nodes (supply and demand) exist with no overlap. Such a case is referred to as a totally unimodular matrix with bipartite qualities (Yannakakis, 1985:280). This type of matrix always yields an integer solution, whether in the relaxed LP condition or not. When the decision variables in location problems take on fractional quantities, Revelle and Swain (1970) recommend using a branch-and-bound algorithm for finding the optimal integer solution. Also, Lagrangean relaxation has been shown to yield success in such applications (Daskin, 1995). Lagrangean relaxation produces an upper and lower bound in which the relaxed objective function value will fall. Although the pmedian problem represented a major milestone in location analysis, the problem made critical assumptions that were addressed by the fixed charge location problem (FCLP).

Balinski's (1965) FCLP relaxes the following three assumptions of the p-median problem: 1. Each potential site has the same fixed costs for locating a facility at it; 2. Facilities that are being sited are uncapacitated; 3. One knows how many facilities should be opened (Current et al., 2002:91). The objective of the FCLP is to minimize total facility and transportation costs. By accomplishing this objective, the model determines the number of locations, location of facilities, and assignment of demand. The FCLP

requires single sourcing of demand. The FCLP has the flexibility to be solved capacitated or uncapacitated. The uncapacitated version involves relaxing the single sourcing constraint. Efroymson and Ray (1966) develop an integer programming method of solving the problem using a Branch and Bound Algorithm. Daskin (1995) recommends solving the uncapacitated FCLP with Lagrangean Relaxation or by the Add or Drop Algorithms and solving the capacitated FCLP with Lagrangean relaxation. The Add and Drop Algorithms follow the same principles as the Greedy Algorithm, but work from different sides of the total cost curve. Although the p-median problem and offshoot problems like the FCLP drove much of the location-allocation problem solving innovation in the classic phase, location experts learned that the models were limited by the types of objectives that could be represented in application.

The two major model innovations realized in this period, responding to the need to formulate models addressing maximum distance objectives, were the p-center or minimax problem, and the set covering problems (location set covering problem and the maximal covering location problem). The objective of the p-center or minimax problem is to minimize the maximum distance between a demand and the nearest facility to the demand. Essentially, the problem uncovers worst case scenario. The p-center or minimax problem was first developed by Hakimi (1964, 1965). The p-center problem can be solved with several variations. The vertex p-center problem restricts the location of facility sites to the nodes of the network while the absolute p-center problem allows facilities to be located along the arcs (Current et al., 2002:86). Each can be solved capacitated or uncapacitated. Although the p-center or minimax solution often produces

the optimum solution for minimizing worst case distance, it does little to address limited resource constraints.

The location covering problems do a good job of handling resource constraints, because both problems locate facilities within a critical distance to demand nodes. The location set covering problem (LSCP), developed by Toregas et al. (1971), determines the minimum number and location of facilities within a specified distance or time constraint from the demand sites. Essentially, the problem solution gives the minimum number and locations of facilities to cover all of the demand. The optimal number of facilities is determined endogenously, or within the model itself. The LSCP allocates each demand node to one facility. Demand is not always allocated to the closest facility. For example, this can occur if two different facilities fall within the maximum distance constraint of a demand node, but the further away of the two must be selected to cover a more isolated demand node. Hence, if the closer facility is not needed to cover any other demand node, then the second best facility must be chosen because it is capable of covering both demand nodes. Just as there are many combinations of ways of covering demand in the LSCP, there are also different methods for solving the problem.

The LSCP can be solved by using linear programming optimization, matrix row reduction, a combination of both, or cutting planes (ReVelle and Williams, 2002:309). The linear programming relaxation of the traditional set covering problem often results in an all integer solution (Current et al., 2002:85). Similar to the p-median problem, this usually occurs when the matrix is totally unimodular with bipartite qualities. Also, as in the p-median problem, this matrix always yields an all integer solution. However, in some instances, the LP relaxation of the LSCP results in a fractional solution. When, this

occurs, Current, Daskin, and Schilling (2002) recommend using a Branch and Bound algorithm to obtain an all integer solution. Daskin (1995) provides a thorough discussion of matrix row reduction rules for this situation. Using a combination of LP and row reduction begins with row reduction to reduce the size of the coverage matrix and ends with LP or a relaxed LP in conjunction with the branch and bound algorithm.

Conversely, the maximal covering location problem (MCLP), developed by Church and ReVelle (1974) exogenously restricts the number of facilities located by a pre-determined fixed number, but maximizes the amount of demand that can be covered within the desired or critical distance. Mandatory closeness constraints can also be included in the MCLP. Figure 1 presents a comparison of three of the classic models.

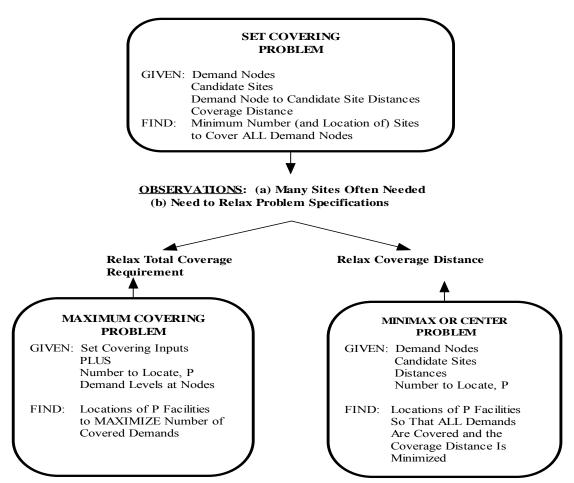


Figure 2.1. Relationships among the set covering, maximum covering, and center problems (Daskin, 1995)

When solving the MCLP for the exogenously determined number of facilities, these types of constraints locate a facility within a mandatory distance. Unlike the LSCP, the MCLP does not always cover all of the demand. The MCLP can be solved by using heuristics such as the GREEDY Algorithm or by LP formulation. The LP formulation will obtain globally optimal solutions (Church and ReVelle, 1974:107). A relaxed LP will occasionally produce fractional answers. Church and ReVelle recommend resolving the fractional problem through inspection or by utilizing the branch and bound algorithm. Although the classic phase of location analysis produced many new location analysis techniques, the contemporary phase took analysis to more realistic proportions.

The Contemporary Phase

The contemporary period of location allocation modeling is characterized by the involvement of a greater number and diverse group of experts in location modeling. This contrasts the classical period in that classical modeling research was mainly conducted by operations researchers and mathematicians. Subsequently, the classical period researchers were mainly concerned with algorithmic efficiency and the mathematical properties of optimal solutions (Ghosh and Rushton, 1987:5). The incorporation of more user experts in the contemporary phase, coupled with the greater use and advances of the high speed digital computer, has allowed the invention of models capable of effectively demonstrating complex system behavior. Additionally, the contemporary phase has seen the re-tooling and greater use of some of the classical models to allow them to do more. Three of the basic models developed in this period are the p-dispersion model, the maxisum-location problem, and the hub location problem. While two of the newer models are location-routing models and facility location-network design models.

Unlike previous models which try to locate facilities closer to demand, the p-dispersion model and maxisum-location problem are designed to put facilities farther away from other facilities and demand respectively. Collectively, the two models are known as the obnoxious location models. The objective of the p-dispersion is to locate p-facilities, maximizing the distance between the closest two open facilities in the network. Only new facilities are considered. "Unlike the p-median and p-center problems, there are no demand nodes and no allocation of nodes to other nodes in the p-dispersion problem" (Kuby, 1987:315-316). The p-dispersion problem is usually solved by using mixed integer linear programming, branch and bound algorithms, or partial enumeration

methods. Potential applications of the p-dispersion problem include the locating of military bases for strategic dispersion purposes, locating of ammunition dumps, and the siting of franchises to prevent cross cannibalization of markets (Current et al., 2002:89). Although the p-dispersion model seeks to disperse facilities for strategic purposes, the maxisum problem spreads out facilities for undesirability reasons.

The maxisum problem, Church and Garfinkel (1978), seeks to locate a discrete number of facilities to maximize the population or demand weighted distance between the population centers and the sites. Ultimately, this maximizes the sum of distances between open facilities. "The maxisum dispersion problem is related to the p-dispersion problem (maximin) in the same way that the p-median problem (minisum) is related to the p-center problem (minimax)" (Kuby, 1987:321). In contrast to the p-dispersion problem, it is difficult to solve the maxisum problem by using branch and bound algorithms because an integer solution must be found on each branch before it can be pruned. Therefore, the maxisum problem is usually solved by Lagrangean relaxation, network flows, and sophisticated data structures (Erkut and Neuman, 1989:284).

Common applications for the maxisum problem are locating prison facilities and landfills. Not only are the obnoxious facility models a contemporary trend in location modeling, but so is the hub location problem.

Hub location problems have received a great deal of attention in recent literature because of the greater reliance on transportation and communication networks. The first hub location problem was the hub median problem developed by Golden (1969), who took Hakimi's (1964, 1965) p-median problem and applied a node optimality property. Hub networks allow service to be provided to customers via a smaller set of links

between origins/destinations and hubs, and between pairs of hubs (Campbell et al., 2002:373). The objective of most hub location models is to minimize total cost with respect to distance. O'Kelly (1987) used a quadratic integer programming method to site airline hubs. Common hub location problems analogous in name and objective to the classic location problems are: the p-hub median problem, uncapacitated hub location problem, p-hub center problem, and hub covering problems (Campbell et al., 2002:375). Although hub location problems are similar to traditional location models, they do present distinct differences.

One of the most significant differences between traditional location problems and hub location problems is that hub location problems permit single and multiple sourcing. Hubs can also perform dual roles, such as switching or consolidation operations. In performing these respective operations, hubs can redirect and combine flows. Hub location problems are solved by a number of different linear programming and relaxation methods, enumeration algorithms, and a wide range of heuristics. Although hub location problems have received a great deal of attention in contemporary location literature, the trend is moving toward more location-routing models.

Location-routing models differ from the basic models because the basic models assume that demand is served directly from a facility. In location routing models, the overall effectiveness of facility location depends not only on the demand weighted distance, but also upon the vehicle route efficiency. Daskin (1995) highlights five fundamental decisions in location-routing problems: 1. How many facilities to locate; 2. Where to locate the facilities; 3. How to allocate customers to the facilities; 4. Which customers to assign to which routes; 5. In what order to route vehicles to customers.

Location-routing models are commonly used in less-than-truckload (LTL) shipping, sanitation, and distribution industries, and are usually solved subject to capacity and or cost constraints. Due to their complexity, Perl and Daskin (1985) recommend heuristics to solve these types of problems. Laporte, Norbert, and Taillefer (1988) go through an effective heuristic that is a derivative of the Traveling Salesman Problem (TSP). Although location-routing models have an assumed network, facility location-network design models require that the network be determined.

In facility location-network design models, one must determine the location of the arcs or network as well as the location of the facilities. "Examples of such problems include the design of subway or rail systems, electricity distribution systems, and computer networks" (Current et al., 2002:96). Also, the design of airline hub and spoke systems follows this type of model because the connection of non-hub airports must be decided as well as the location of hubs. In most cases, the objective(s) in the overall problem is to minimize both facility and network costs. For instance, Current and Schilling (1989) formulated the Covering Salesman Problem (CSP), which introduces location set covering to the TSP with the ultimate goal of minimizing costs. Due to the large number of solution possibilities, facility location-network design problems are usually solved through heuristics, an example of which can be seen in Current and Schilling (1989). The COVTOUR heuristic incorporates the solution procedures for the Traveling Salesman Problem and the LSCP (Current and Schilling, 1989:210). Ultimately, most location models take on or use common characteristics and objectives of the basic models. Similarly, models also contain common taxonomies.

Common Location Modeling Taxonomies

Location models may be classified in many different ways. Models can be classified based on the nature of the inputs and the number of products to be provided by the facilities. Furthermore, models can be classified by sourcing, number of objectives, capacity, elasticity of demand, or the sector of the economy that models are designed to accommodate (private versus public). Finally, models can be classified based upon the type of distances that are used between locations in the model. This section develops the classification criteria commonly found in many location problems and models.

Planar versus Network versus Discrete Location Models

One of the key differentiators in location modeling is how candidate facilities and demands are represented (Daskin, 1995:10). In planar location models, facilities and demands can occur anywhere on a plane. Thus, there are an infinite number of facility location possibilities. Conversely, network location models only permit travel between demand sites and facilities on arcs of a network. Demand locations can occur at nodes or anywhere on the links of the network depending on the model formulation. In contrast, discrete location models only allow sites to be located at a specific group of sites or nodes.

Discrete location models allow for the use of arbitrary distances between nodes. As such, the structure of the underlying network is lost. However, by removing the restriction that the distances between nodes be obtained from an underlying network, the more general class of discrete location models allows a broader range of problems to be modeled. (Daskin, 1995:10-11)

Discrete location models also generally start with a set of candidate facilities as opposed to locating anywhere on a plane or on a network. Although this type of taxonomy

addresses the nature of location selection as a whole, it does little to address how distances are characterized in the network.

Distance Metrics

Location models can be characterized by the method of measuring distance used in the model. Distance in location models can be represented in several ways: 1. By Euclidean or straight line distances; 2. By geographic distances (using latitude and longitude); 3. By rectilinear or Manhattan distance; 4. By routing factors; or 5. By actual transportation distances from a road or rail system. Geographic representation is based on the distance between two sets of latitude and longitude coordinates and often takes into account the curvature of the earth through the use of trigonometric formula for computing great circle distances. Euclidean, vector, or straight line distance is figured as a straight line between two points and was the most common technique in early location modeling. Rectilinear or Manhattan distances are computed as travel distance on a grid (i.e., north-south and east-west travel). This method is commonly used in urban travel networks. Distances by routing factors often take Euclidean or rectilinear distances and multiply them by a routing factor to simulate actual transportation distances. Not only are location models classified by the type of distance metrics used, but they are also differentiated by the number of facilities to locate.

Number of Facilities to Locate

Location problems can be classified by how the number of facilities is computed.

In some models, the number of locations is exogenously determined by the user. This is

true in the p-median, p-center, and maximal covering problem. However, in the LSCP and FCLP the number of facilities is endogenously determined because the number of facilities is the model output. When the number of facilities is to be exogenously determined, the problem must also be differentiated as a single site or multiple site model (Daskin, 1995:13).

Static versus Dynamic Location Problems

Location models can be classified and solved as static or dynamic. Static model inputs do not depend upon time. Static model inputs are essentially a snap shot in time of a representative period. Since static models are easier to handle, most location models are constructed statically, but the actual problem is usually dynamic. Dynamic model inputs vary with time. "In dynamic problems, we are concerned not only with the question of where to locate facilities, but also with the question of when to invest in new facilities or to close existing facilities" (Daskin, 1995:13). The current trend in location literature is leaning toward the development of more advanced dynamic models. Current et al. (1998) recognize two classes of dynamic models: implicitly dynamic and explicitly dynamic. Implicitly dynamic models are designed such that all of the facilities will be opened one at a time and will remain open throughout the planning timeframe (Current et al., 2002:98). Conversely, explicitly dynamic models are designed for problems where facilities will be opened and closed over time (Current et al., 2002:98). The opening and closing of facilities can correspond to changes in problem parameters over time.

Deterministic versus Probabilistic Models

Inputs to location models can be classified as deterministic or probabilistic. In deterministic models, the inputs for demand, supply, and time are certain. Inputs to

probabilistic models are based on a probability distribution and can be subject to some uncertainty. Huff (1962) built a probabilistic gravity model, which incorporated the probability that a consumer would visit a certain shopping center. Probabilities can be based on market research in a retail setting or by utilizing historical data to set probabilities in a host of other scenarios.

Single- versus Multiple-Product Models

Location models can be differentiated by the number of products and services offered by all or some facilities, and thus, will drive which consumer demand could be serviced by certain locations. In single product models, a single homogenous product or service is offered by all facilities. Also, demand for product and service types is assumed identical in many of the classic location models. Multiple product models are characterized by different products and service across different locations as well as different demands for products and services in respective demand segments.

Warszawski (1973) formulated the original multi-commodity or multi-product plant location problem. The algorithm seeks the least cost sites for manufacture and distribution of different products, where each site can produce at most one product. For a more current examination of single and multiple product models, the reader is referred to ReVelle and LaPorte (1996), who compile different single and multiple product algorithms in the context of the plant location problem.

Private versus Public Sector Problems

Problem differentiation between private and public sector models lies in profit maximization. Generally, private sector models are geared at minimizing costs and

maximizing investment dollars. Public sector problems must often take into account non-profit or monetary related objectives such as equity of treatment between different demand sectors. In many cases, public sector models must incorporate the political process or objectives between competing parties. For a review of private and public sector location models, the reader is referred to ReVelle et al. (1970). Although different in taxonomy, private sector tools can be successfully applied to public sector problems.

Marianov and Taborga (2001) demonstrated how a public service health clinic could compete with private providers to reduce the required state funding or subsidy. Additionally, many of the location tools were developed to solve public sector problems. Toregas et al. (1971) used the location set covering problem to show how to optimally locate emergency vehicles. Church and ReVelle (1974) developed the Maximal Covering Location Problem based on siting a fixed number of facilities subject to a maximal service distance. The concept of maximal service distance is well suited to the public sector operations of fire stations and ambulance dispatching facilities. Eaton et al. (1981) used this concept in siting health clinics, while Moore and ReVelle (1982) extended the service distance concept to hierarchical health services.

Single-versus Multiple-Objective Problems and Models

As is the case in static versus dynamic model formulation, most location models are formulated with a single-objective, but the problem under examination remains multi-objective in nature. This often occurs because problems have competing stakeholders. To identify tradeoffs, single objective models can be run with a range of inputs (Daskin, 1995:16). Solving multiobjective problems generally involves one of two approaches: generating techniques and preference-based techniques (Current et al., 2002:97).

Preference-based techniques involves weighting or ranking of objectives and solving with the rank-ordered objectives. "In general, generating techniques identify the pareto optimal siting configurations from which decision makers select the ones that they prefer" (Current et al., 2002:97). Schilling and others (1980) developed a Multiobjective Facility Location Problem (MOFLO) that takes into account the trade-offs and alternatives such as minimizing costs and maximizing coverage inherent in locating fire protection equipment. Much of the contemporary location analysis research is leaning toward the multiobjective models because most location problems are multiobjective in nature. The reader is referred to Current et al. (1990) and Erkut and Verter (1995) for a comprehensive review of multiobjective facility location problems.

Elastic versus Inelastic Demand

Location models can also be classified by their type of demand. Inelastic models treat demand independent of the level of service. For example, if a person needs emergency surgery he or she generally would not inquire about the cost. Inelastic demand is illustrated in Toregas et al. (1971) as well as Church and ReVelle (1974). Elastic models treat demand as dependent on service levels. Services and stores offered by shopping centers can have a profound impact on whether consumers will patronize them or not. Although most real world location problems exhibit some degree of elasticity of demand with respect to service, in most cases location problems are treated as having inelastic demand (Daskin, 1995:16). However, the use of elastic demand in location modeling can be seen in Perl and Ho (1990) and Kuby (1989). Perl and Ho compare the location behavior under elastic and inelastic demand, and investigate the

effects of the demand function on facility location. Kuby's model maximizes the number of firms that can exist in a market using Losch's central place theory.

Hierarchical versus Single-Level Models

In some instances, location models are hierarchical. This trait is characterized by a hierarchy of flows between facilities. For example, before a patient is transferred to a cardiac specialist he or she would probably be seen by a medical practitioner at a different facility. Hierarchical models are commonly used in production layout design models where parts must complete a prescribed number of steps before proceeding to a specific facility or station. Typical location models are single-level models, which are characterized by one-stop shopping types of service. Hierarchical model research is presented by Moore and ReVelle (1982), who developed a nested, parallel hierarchical covering model for locating medical services. Also, Tien and El-Tell (1984) developed and applied a hierarchical model and applied it to the healthcare system of a 31 village region of Jordan.

Capacitated versus Uncapacitated Models

Facilities in location models may be unrestricted or restricted on the amount of demand that they can fill. In uncapacitated models, facilities are treated as having unlimited capacity. Capacity is considered to be unlimited in the LSCP, MCLP, p-median, and p-center models. This is respectively illustrated in Toregas et al. (1971), Church and ReVelle (1974), and Hakimi (1964, 1965). The distance or cost constraints in these models superficially impose capacity restrictions. In capacitated models, facilities are limited in the amount of demand that they can serve. Also, transportation routes and demand amounts at locations can be capacitated. Mirzaian (1985) used the

capacitated concentrator location problem (CCLP) to assign telecommunications terminals to concentrators with fixed capacities. Models with finite capacity at different facilities often drives a key modeling decision whether to allow multiple or single sourcing.

Single versus Multiple Sourcing Models

In single sourced models, demand is served by one source subject to the service criteria set forth in the model. Most location models are single sourced. Multiple sourcing models allow demand to be served in all or part by a number of different facilities. Cooper (1967) recognized and modeled a multiple source approach where the demand at a single location can be supplied from one or more sources. Bell and Mullen (2003) used a multiple sourcing approach to supplying munitions based on stochastic demand. Multiple sourcing is often used in conjunction with capacitated facilities, where a portion of demand is served by one facility and the remainder is served by a different location(s). Allowing multiple sourcing also heightens the complexity of location models because the number of possible solutions is dramatically increased. Not only is it important to recognize the taxonomy of location models, but it is also paramount that the correct decision technique is selected to solve the problem.

Solution Approaches for Location Models

For years location scholars have employed a variety of techniques to solve location models. The two most common techniques are mathematical optimization and heuristics. Mathematical optimization involves using mathematical formulae to find the best answer to a problem. Heuristics entail using a "rule-of-thumb" or common sense

algorithm to find a good solution. Although many examples of these types of techniques have been mentioned in previous sections of this chapter, this section explores them in further detail. Table 2.3 shows common advantages and disadvantages of optimization and heuristics. The optimization information was obtained from Powers (1989) and the information on heuristics was obtained from Ballou (1989).

Table 2.3. Advantages and Disadvantages of Optimization and Heuristics

Advantages	Disadvantages
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Optimization	 Guaranteed best possible solution given assumptions and data Can accurately handle all forms of costs (variable and fixed) Creative solutions not considered before can be uncovered Permits more efficient analysis of problems (economizes data efforts) Often results in significant 	1. Can assume away the problem 2. Optimization cannot be used for full range of logistic problems 3. "Black box" syndrome (some managers do not understand mathematical algorithms behind technique) 4. Optimal solutions do not prescribe operating rules for implementation
Heuristics	cost savings 1. Allow optimal or near optimal solutions 2. Solution time is reduced 3. Solution satisficing (close solution good enough) 3. Best to use when resources are constrained 4. Heuristics can do a better job of accurately describing the problem	5. Tough to use in larger models 1. Solution is not optimal 2. Do not handle capacities and fixed costs well

The selection of one of the aforementioned techniques is often driven by model size or complexity. Specifically, some models are too large or too complex to be solved by optimization methods because it could significant amounts of time and as well as large amounts of computational resources to solve them. This occurs because basic location models such as the p-median and p-center problems are often classified as nondeterministic polynomial (NP)-hard (Garey and Johnson, 1979). All NP-hard problems can be solved through mathematical optimization; however, some problems require too much time to solve utilizing this method, therefore, a heuristic is more

desirable. Not only can location problems be classified as NP-hard, but they can also be classified as NP-complete.

NP-complete problems are a class of computation problems for which no efficient solution algorithm has been found. If a problem is NP and all other NP problems are polynomial-time reducible to it, the problem is NP-complete. One of the most widely known NP-complete problems is the Traveling Salesman Problem (TSP). Thus, problems in the NP-complete class cannot be solved by optimization methods. Similarly, many problems that are NP-hard must be solved by using a heuristic as opposed to mathematical optimization. Although many location problems are frequently solved through the use of heuristics, optimization provides the optimal solution whereas a good heuristic gives a near optimal, and in some instances, an optimal one.

Optimization Methods

The two most common methods of optimization are complete enumeration and mathematical programming. Complete enumeration involves a person or computer looking at every possible combination of variables in a problem to arrive at the optimal solution (Powers, 1989:107). This method is cumbersome and generally works best for smaller problems. As problems get larger, mathematical programming methods are often used because complete enumeration becomes too difficult. Common types of mathematical programming are linear programming, integer programming, non linear programming, and mixed integer-linear programming.

Linear programming (LP) involves solving problems optimally by using linear objective functions and linear constraints. Ragsdale (2001) outlines five steps in formulating an LP:

1. understand the problem; 2. identify the decision variables; 3. state the objective function as a linear combination of the decision variables; 4. state the constraints as linear combinations of the decision variables; 5. identify any upper or lower bounds on the decision variables. (Ragsdale, 2001:21-22)

Once the linear programming problem is formulated it can then be solved. The three common ways to solve an LP problem are: using level curves, enumerating corner points, and utilizing a spreadsheet solver. The first two methods have limited practical use because they can be used only in instances where there are two decision variables (Ragsdale, 2001:25). Many LP problems are solved with a spreadsheet solver package such as Solver™ for Microsoft Excel® or LINDO. Spreadsheet solver packages apply common solution algorithms to LP problems that are capable of solving problems with multiple decision variables. When solving an LP, several special conditions can arise in the solution process: 1. Alternate optimal solutions; 2. Redundant constraints; 3. Unbounded solutions; 4. Infeasibility (Ragsdale, 2001:33). Alternate optimal solutions will be discussed in Chapter 3. Redundant constraints have no bearing on the solution. The last two conditions prevent one from solving the LP problem. An unbounded solution suggests formulation errors. The most common way of dealing with infeasibility is to relax or adjust the constraint causing the infeasibility. It must be mentioned that if the original problem is infeasible, then, relaxing the constraint creates a new LP.

In some instances, the optimal values of the decision variables must take on integer values. This method is known as integer linear programming (ILP). Many of the classical location problems can be solved as ILP models. ILP models are formulated in the same manner as LP models. ILP problems can be solved through LP relaxation, branch and bound algorithms, and through using a spreadsheet solver program. LP

relaxation does not guarantee an optimal integer solution, because the LP solution must often be rounded. Branch and bound algorithms solve ILP problems by solving a series of LP problems called candidate problems (Ragsdale, 2001:237). While ILP problems require that the decision variables take on integer values, mixed integer linear programming (MILP) problems can have integer and non-integer decision variable values. All of the formulation and solution methods previously discussed apply to MILP problems.

The final mathematical programming method is non-linear programming (NLP). NLP problems contain objective functions and constraints that cannot be modeled adequately using linear or straight-line functions (Ragsdale, 2001:336). NLP problems can be formulated and solved much like LP problems; however, the mathematical procedures behind solving NLP problems are different (Ragsdale, 2001:336). Spreadsheet solver platforms make this difference almost transparent. LP, ILP, and MILP problems are generally solved using the Simplex algorithm in Solver™ for Excel® where NLP problems are solved in Solver™ with the generalized reduced gradient (GRG) algorithm. For a complete explanation of how the Solver™ algorithms arrive at optimal solutions the reader should consult Ragsdale (2001), Chapters 4 and 8.

Each of the previously discussed methods has been proven mathematically to arrive at the best achievable or optimal solution (Powers, 1989:107). Selection of one method over the other depends upon the structure and objective(s) of the problem under investigation. The optimal solution can be refined to two different types of solutions: local optimal and global optimal. A local optimal solution is a solution that is better than any other feasible solution in its local or immediate vicinity (Ragsdale, 2001:339). A

global optimal solution is the best possible solution to a problem. A local optimal can also be a global optimal solution. While optimization guarantees an optimal solution, good heuristics can often produce near optimal results in a fraction of the time.

Heuristics

The use of heuristics has increased with the evolution of location modeling, because more realistic modeling makes optimization more difficult due to problem size. As previously stated, the goal of a heuristic is to find the best solution possible. In many instances, the solution found is not optimal. Three common types of heuristics used in location modeling are greedy heuristics, improvement heuristics, and Lagrangean relaxation

One of the earliest documented heuristics is the greedy heuristic. Greedy heuristics seek to choose locations that have the greatest impact on the value of the objective function. The two most common greedy heuristics are the Greedy-Add Algorithm developed by Kuehn and Hamburger (1963) and the Greedy-Drop Algorithm. The operation of both greedy heuristics has been covered in the Classic Phase of Location Modeling Section in this chapter. "While both the Greedy-Add and the Greedy-Drop heuristics are effective at identifying a feasible solution with modest computational effort, neither can be relied upon to consistently produce good solutions" (Current et al., 2002:102). Improvement heuristics seek to correct this problem because they begin with a feasible solution.

Improvement heuristics have been around as long as greedy heuristics. Three of the most common improvement heuristics are the Neighborhood Search Algorithm,

Interchange Algorithm, and the Tabu Search method. One of the first improvement

heuristics is Maranzana's (1964) Neighborhood Search Algorithm. The Neighborhood Search Algorithm assigns demand to the nearest facility and evaluates each neighborhood around the facility for the best solution to serve the demand (Current et al., 2002:102). Then, the process is repeated. The iterative process is continued until no changes can be made to facility sites or neighborhoods. While the Neighborhood Search Algorithm evaluates the surrounding neighborhood of a facility for objective improvements, the Interchange Algorithm substitutes open with unused sites.

The Interchange or Exchange Algorithm developed by Teitz and Bart (1968) exchanges open sites with unused sites to improve upon the solution. A complete description of the performance of the Interchange Algorithm is given in the Classic Phase of Location Modeling Section of this chapter. The drawbacks to the previously mentioned improvement heuristics lie in their ability to get "stuck" on local optima solutions (Current et al., 2002:103). To combat local optimal solutions researchers have employed the use of modern metaheuristics such as the Tabu Search Algorithm and Simulated Annealing.

Tabu Search Algorithms guide the application of core search heuristics (Neighborhood Search and Interchange) by inhibiting certain moves or exchanges to get the algorithms to explore other regions of the solution space rather than just the area surrounding a local optima solution (Current et al., 2002:103). Defining which moves or searches to restrict is central to the successful application of the Tabu Search Algorithm. Good examples of application of the Tabu Search Algorithm to location problems can be found in Klincewicz (1992), who applied Tabu Search to the p-hub location problem and Rolland et al. (1997), who used Tabu Search in the p-median problem.

Simulated Annealing was first proposed by Metropolis et al. (1953) to simulate the process of annealing molten metals to solve combinatorial optimization problems. The heuristic search procedure of Simulated Annealing uses a cooling schedule to control the process of accepting an inferior solution in the overall annealing process. Bell and Mullen (2003) used this method to solve a munitions distribution problem given stochastic war time scenarios in the European Theater. Although none of the previously covered heuristics in this section can guarantee an optimal solution nor specify a range in which the optimal solution lies, Lagrangean relaxation addresses the bounds on an optimal solution through the use of a Lagrange multiplier.

Lagrangean relaxation is an optimization based heuristic that can be used to solve large location problems. Essentially, the objective function of an optimization formulation remains; however, one or more constraint(s) are relaxed by multiplying them by a Lagrange multiplier. Lagrange multipliers are often found through using a search heuristic (Current et al., 2002:105). After the constraint(s) are multiplied by the Lagrange multiplier, the new constraint is brought into the objective function. Then, the relaxed problem is solved and the decision variable values become lower bounds for the optimal solution and are used to compute the upper bound for the objective function (Daskin, 1995:122). The original solution to the relaxed problem is compared to the bounds to see which of the relaxed constraints are violated (Daskin, 1995:122). Then, the Lagrange multiplier is updated and the problem is resolved iteratively until the relaxed constraints are not violated. Current et al. (2002) recommend using subgradient optimization to update Lagrange multipliers. Daskin (1995) shows how Lagrangean relaxation can be successfully applied to the FCLP, MCLP, and the p-median problem.

Also, the reader should consult Fisher (1985) for a methodology of the application of Lagrangean relaxation. Ultimately, location analysis modeling techniques contribute to the effective application of location analysis.

Applications of Location Modeling

As previously stated, the bulk of the location analysis literature is directed toward developing better and more efficient modeling techniques and solution algorithms. This does not mean that location modeling has not been used successfully for real world applications; it means that a limited number of location modeling case studies have been published.

Many successful case study applications of location modeling are not published because: 1. applications frequently employ existing models and solution techniques; 2. specific applications are frequently analyzed by consultants and planners; two professions that are not compelled to publish; 3. private sector advances in location modeling are often viewed as proprietary. (Current et al., 2002:82)

Some location problems such as locating ambulance depots have an extensive documentation in the previous literature; however, this is not the norm. Although the location modeling application literature is not as robust as the documentation on location algorithms or solution techniques, there are industries where location analysis has been used and the results published.

Commercial Applications of Location Modeling

Commercial applications of location models have a documented wide range of applications and positive results in the previous literature. Such applications range from the siting of airline hubs in O'Kelly (1987) to locating electric power generating plants by

Cohon and others (1980). Schilling and others (1980) used a multiobjective formulation of the MCLP to locate fire protection equipment and facilities for the City of Baltimore. Similarly, Branas and others (2000) developed The Trauma Resource Allocation Model for Ambulances and Hospitals (TRAMAH) to optimally locate emergency trauma centers for the State of Maryland. TRAMAH achieved a 5.17% increase in the availability of trauma centers within a 30 minute response, while reducing the number of required aeromedical depots by six to achieve the current level of service within a 15 minute response (Branas and others, 2002:489). Swersey and Lakshman (1995) used simulation in conjunction with location set covering to determine the number, size, and locations of vehicle emission testing stations in the State of Connecticut, reducing network costs by \$3 million. While the previously mentioned applications are not all-inclusive, they clearly demonstrate that location analysis has been applied successfully to the commercial sector. For a thorough list of common location modeling applications in the literature the reader is referred to Current (2002). Although commercial applications of location analysis are well documented in previous literature, this is not the case for military problems.

Military Applications of Location Modeling

The previous literature is extremely limited in the practical application of location analysis techniques to military problems. Skipper (2002) used multiple objective linear programming (MOLP) to analyze optimal hub locations in the United States European Command. Specifically, Skipper incorporates Bryan and O'Kelly's (1999) hub location quadratic single assignment model with MOLP to determine the optimal hub location based on cost and time. The hub location model was originally formulated as a quadratic

integer program by O'Kelly (1987) and applied to the location of airport hubs in the U. S. Also, Garcia (1995) developed a multiple-cover, multiple-location allocation heuristic to optimally locate reparable support equipment and repair facilities for the Air Force given historic demand data. Garcia's research also compared his solution with the current Air Force depot configuration. Finally, Bell and Mullen (2003) used location analysis to optimally position munitions given stochastic wartime scenarios. Not only has location modeling been used for practical applications in the military, but it has also been used to evaluate the effectiveness of positioning decisions in previous conflicts.

ReVelle and Rosing (2000) developed a set covering deployment problem (SCDP) and maximal covering deployment problem (MCDP) to demonstrate how the Emperor Constantine (Constantine the Great) could have more effectively deployed his armies to defend the Roman Empire in the event of a two-front war. This application shows promise to American military strategy because of its ability to efficiently allocate troops given military objectives subject to resource constraints. It is this researcher's opinion that location modeling has been used on many more occasions in military applications than is documented in the literature; however, the modeling efforts and results are proprietary and have not been published. Ultimately, the recent war on terrorism, downsizing of the military, and declining budgets is increasing the importance of location analysis to the military.

Summary

This chapter traced the history of the strip alert network in the United States from the Cold War period to present. The purpose of the history was twofold: 1. to review the

objectives of the strip alert network as it pertained to national defense and homeland security; and 2. to review some of the previous strip alert sites, alert postures, and ways of responding to threats. An evolution of location modeling techniques was presented to so that the proper selection of modeling technique could be made as it applies to the objectives of the modern strip alert network. Also, common solution techniques used in solving location problems were presented to aid in selection of the best method to solve the problem of locating strip alert assets optimally within the guidelines of the modeling technique selected. Finally, common applications of location modeling techniques were presented to demonstrate the wide range of problems that the location analysis techniques are capable of modeling as well as ensure that location analysis has not been applied to optimizing a fighter strip alert network in previous research.

III. Methodology

Introduction

This chapter discusses the framework for the analysis utilized in this research.

Location analysis methods and solution techniques used in this study and their relevance are presented, and sources of data and methods of retrieval are introduced. Then, network objectives and critical model parameters are discussed, followed by network operation assumptions, and location modeling method(s) selection and presentation.

Additionally, solution technique selection is presented to show how the research question of optimal strip alert site location in the CONUS will be answered. Formulations used for distance calculations between candidate sites and areas of interest are presented to assist in building location distance and coverage matrices to enable the use of Microsoft Excel's[®] Solver™ Add-In to construct the necessary integer programming (IP) models. Finally, the models constructed for analysis are presented to lay the foundation for the results and sensitivity analysis in Chapter 4.

Data

Data was provided to this study by personnel at the ACC Department of Homeland Security and First Air Force Air Operations Center (AOC). As mentioned in Chapter 1, ACC is the primary force provider for the strip alert network and First Air Force is charged with executing the alert network on behalf of the CONUS portion of NORAD. The overall strip alert network objectives were determined by the ACC Department of Homeland Security. They determined their relative importance in the

post-11 September strip alert network as well as identified the critical model parameters to aid in location modeling method and solution technique selection. The data obtained from the interviews appear in the next two sections.

Objectives of the Strip Alert Network Post-11 September

Personnel at the ACC Department of Homeland Security and the First Air Force AOC were interviewed to determine the objectives and their relative importance in the overall alert network. Personnel indicated the following desired objectives of the strip alert network post-11 September:

- 1. Minimize aircraft response time.
- 2. Cover all areas of interest with at least one alert site.
- 3. Minimize the number of strip alert locations.
- 4. Minimize overall or average distance per network location.
- Minimize the maximum travel time for an aircraft at any location in the network.

Minimizing the required number of alert sites to cover all of the areas of interest is the first or overarching requirement. All other objectives are considered equally important in the overall network. Aircraft response time is the amount of time (notification to arrival) that it takes an aircraft to fly from a candidate site to an area of interest. In accordance with First Air Force policy, areas of interest are those areas that require protection in the interest of National Security. Finally, a strip alert location is a candidate alert site which meets the criteria for operational capability. After outlining the objectives of the overall network, personnel were asked about the specifics of network operation to determine the critical model parameters.

Critical Model Parameters

Critical model parameters are the underlying operational requirements of the model permitting effective modeling technique and solution method selection. These parameters include aircraft type, launch, and operating characteristics as well as candidate site requirements, the list of the areas of interest, and response requirements to the areas of interest. The subsequent paragraphs detail the information obtained in the interviews to build the critical model parameters.

In order to be considered a suitable candidate for a strip alert site, the following two criteria must be met:

- The candidate must be an existing CONUS joint use airfield. Joint use means
 that military aircraft currently operate out of the airfield. This can include any
 branch of the armed forces as well as any component (Guard, Active Duty, or
 Reserves).
- A candidate site's runway must exceed a minimum length. This length was determined by the ACC Department of Homeland Security and is considered proprietary.

Once the criteria were established, candidates were identified through consulting the *DoD Flight Information Publication (Enroute)* dated 10 July 2003 to 4 September 2003. A list of 202 suitable candidates was found. Each site was assigned a specific number to facilitate identification. Actual airfield names identified as suitable candidates will not be given in this research due to security considerations but can be obtained from the ACC Office of Homeland Security or the First Air Force AOC with the proper security clearance.

The aircraft utilized in the model are of two types: F-15 and F-16. Different aircraft models and munitions configurations are not considered. A notional 8-minute launch time is for all candidate alert sites, except for site 69. At this particular site, a notional 5-minute launch time is used. Historical launch or aircraft scramble times for existing sites are classified and are not used in this research. Finally, a best case flight time of 9 nautical miles (NM) per minute is used for both aircraft types.

One of the central model parameters is the areas of interest to be protected by the strip alert network. A list of 70 different areas of interest was obtained from the First Air Force AOC. This list was compiled from a variety of sources and is not divulged in this research due to security reasons. Each area of interest was assigned a specific number to facilitate identification. Furthermore, areas of interest are delineated by type. Type I areas of interest require constant strip alert coverage while Type II only require strip alert coverage when requested by NORAD. Response times to each area of interest vary by area type and in some instances, specific area. Table 3.1 outlines the different response requirements.

Table 3.1. Desired Aircraft Response by Area Type and Exceptions

Area Type	Desired Response	Specific Area Exceptions
Type I (Areas 1-27 and 31-	≤ 20 minutes after	Area 13 response time is ≤
69)	notification	12 minutes after notification
Type II (Areas 28, 29, 30,	≤ 12 minutes after	Area 70 response time is \leq
and 70)	notification	20 minutes after notification

Note. Response time includes both launch and flight times in all instances.

The response times listed in Table 3.1 are notional. The actual response times are classified and can be obtained through official channels with the proper security clearance and need to know. After obtaining the desired strip alert network objectives

and the critical model parameters, network operation assumptions are made to simplify the model building process.

Candidate Site Assumptions

Since the objectives of the overall network are response time and coverageoriented as opposed to cost, it is assumed that all candidate sites possess the necessary
infrastructure and support network to support the strip alert mission. Infrastructure and
support includes, but is not limited to, personnel, ground support equipment, airborne
tanker support, and hangaring space. Furthermore, it is assumed that any site considered
meets the necessary explosive quantity-distance requirements for the types of munitions
loaded on the jets. It is assumed that no airspace restrictions exist around any area of
interest and it is also assumed that the number of aircraft placed at any alert site has no
bearing on overall response time. Finally, politics are assumed to play no part in site
selection and that each site has an equal probability of selection. Once site specific
assumptions are made to simplify the operations of the candidate sites in the problem,
assumptions are made regarding the network aircraft operating characteristics.

Aircraft Operation Assumptions

To limit the complexity of the model, it is assumed that the F-15 and F-16 aircraft perform similarly throughout the network. Furthermore, it is assumed that the desired number of aircraft launch and arrive successfully at the required area of interest.

Essentially, this ignores the possibility of aircraft ground or air aborts. Finally, it is inferred that the aircraft launch or scramble times follow historical trends. These

assumptions reduce model complexity because infinite combinations of launch times and aircraft performance data would undoubtedly increase the number of possible network coverage schemes. Subsequently, this would make a heuristic a more desirable solution technique because of its ability handle a greater number of modeling possibilities. After the assumptions were made a solution technique was selected to solve the problem.

Selection of Solution Technique

As discussed in Chapter 2, the two primary techniques available for solving location problems are optimization and heuristics. Mathematical optimization was selected as the solution method in this research because of the overarching objective to find the optimal network configuration. A heuristic does not guarantee optimality. Additionally, mathematically optimal formulae exist in the previous literature that are capable of finding the best solution to the objectives of the problem. Optimization is considered the best method because of optimization's ability to uncover solutions not previously considered, and its efficient analysis characteristics. Subsequently, the decision to use optimization makes mathematical programming the preferred optimization method due to the large number of candidate sites and areas of interest.

The large number of sites in this problem makes complete enumeration time consuming, which would limit the amount of sensitivity analysis that could be conducted. After selection of mathematical programming as the desired optimization technique, it was decided to use integer programming as the mathematical programming method. Integer programming is the best fit because: 1. The formulations of the location modeling methods are already in integer programming format; 2. The areas of interest

cannot be served by multiple sites; and 3. There cannot be fractional demand. After making the critical modeling assumptions and selecting optimization as the preferred solution technique, specific location modeling methods were selected.

Location Modeling Method Selection

Location modeling method selection is one of the most important aspects of any location research effort. After a literature review of existing location modeling techniques in Chapter 2 and the identification of the strip alert network objectives as well as the critical model parameters, the LSCP, the p-median, and p-center methods were chosen. Selection was based on the objectives of the overall network listed earlier in this chapter. The LSCP is effective at fulfilling objectives 2 and 3, while the p-median problem is effective at meeting objective 4 and providing the key input to fulfill objective 1. Finally, the p-center is chosen for its ability to meet objective 5. Additionally, the candidate sites and areas of interest are each a set of discrete locations and all techniques are proven to be effective in discrete location modeling. The suitability of these techniques in meeting the mentioned objectives was discussed in Chapter 2. With the techniques selected, the respective problems were mathematically formulated.

Mathematical Formulation of the Location Set Covering Problem

As indicated in Chapter 2, the LSCP is designed to locate the minimum number of facilities within a distance or time constraint. In this research, the required response time was converted into a critical distance by taking into account aircraft launch and flight time. The distance metric used and computation of the critical distance is discussed later in this chapter. The problem is structured as an integer programming problem. All

facility costs are assumed to be identical in this formulation and are not included in the objective function. Also, the model assumes the alert sites are uncapacitated and single sourcing of demand. As discussed in Chapter 2, the original LSCP was developed by Toregas et al. (1971); however, the formulation used in this research is borrowed from Revelle and Williams (2002). Given that the critical distance between areas of interest and candidate sites is varied in this research, the model requires that an adjustment be made to the maximum allowable distance notation. The notation used is stated as:

i, I = the index and set of areas of interest or nodes;

j,J = the index and set of candidate alert sites or nodes;

 d_{ij} = the shortest distance or time between points or nodes i and j;

 S_{ij} = the maximum allowable distance computed from response and launch times; an alert site located at node j within the standard of the area of interest node i is eligible to serve the area of interest;

 $N_i = \{j | d_{ij} \le S_{ij}\}$ is the set of alert sites j within the critical distance S_{ij} of area of interest i;

 $X_j \in \{0,1\}$ it is 1 if an alert site is located at site j, and 0 otherwise.

The LSCP formulation used in this research is as follows:

MINIMIZE
$$\sum_{j \in J} X_j \tag{1}$$

SUBJECT TO:
$$\sum_{j \in N_i} X_j \ge 1 \qquad \forall i \in I$$
 (2)

$$X_j \in \{0, 1\} \qquad \forall j \in J \tag{3}$$

The objective function (1) minimizes the number of selected alert sites needed to cover each and every area of interest by at least one facility. Constraint (2) requires that each area of interest must be covered by at least one alert site within *S* distance or time units of it. Constraint (3) is the integrality constraint. The LSCP is classified as NP-hard (Garey and Johnson, 1979).

If this problem is solved a priori, there exists a good probability of alternate optimal solutions. Specifically, there is a chance of finding many different combinations of the minimum number of locations capable of covering the demand. For instance, if the LSCP found the minimum number of sites to cover the areas of interest to be 21 out of the 202 candidates, then there is the possibility of there being 8.71⁶⁹⁴ alternate optimal solutions. This is computed by using the following formula known as the combinatorial rule (McClave et al., 2001:129):

$$\frac{N!}{n!(N-n)!} \tag{4}$$

where:

N = number of candidate alert sites; and

n = minimum number of sites needed to cover all areas of interest.

As previously stated, any alternate optimal solution of the LSCP can meet objectives 2 and 3; however, an optimal solution is needed to satisfy objectives 1 and 4 as well. By using the p-median problem in conjunction with the LSCP, the number of alternate optimal alert site configurations is minimized, because there is only one minimum aggregate network distance. Therefore, the minimum number of locations computed from the LSCP will be utilized in the p-median problem. Now that the mathematical

formulation for the LSCP has been determined, the p-median problem that will be utilized in the research is formulated to minimize the aggregate or total alert network distance.

Mathematical Formulation of the P-Median Problem

As discussed in Chapter 2, "the objective of the p-median model is to identify locations for p facilities in some space to serve n demand points so that the total weighted distance (or cost) between the facilities and the demand points they serve is minimized" (Bozkaya, 2002:180). As previously discussed, the number of facilities utilized in this model is taken from the results obtained through the LSCP. While the first formulations of the p-median problem come from Cooper (1963) and Hakimi (1964, 1965), the formulation of Daskin (1995) with a minor adjustment is utilized in this research. The adjustment removes the demand weight multiplier from the objective function, because the demand in this model is assumed equal. The formulation is as follows:

MINIMIZE
$$\sum_{i} \sum_{j} d_{ij} Y_{ij} \tag{5}$$

SUBJECT TO:
$$\sum_{j} Y_{ij} = 1 \qquad \forall i$$
 (6)

$$\sum_{j} X_{j} = P \tag{7}$$

$$Y_{ij} - X_j \le 0 \qquad \forall i, j \tag{8}$$

$$X_j = 0,1 \qquad \forall j \tag{9}$$

$$Y_{ij} = 0,1 \qquad \forall i, j \tag{10}$$

where

 $X_i = 1$ if we locate at candidate site j, 0 otherwise

 $Y_{ij} = 1$ if area of interest i is served by candidate alert site j, 0 otherwise

 d_{ij} = travel distance between area of interest *i* and candidate alert site *j*

P = number of alert sites to be located; taken from the LSCP results.

The objective function (5) minimizes travel distance between the areas of interest and each selected alert site. Constraint (6) requires that each area of interest be served by one alert site. Constraint (7) states that exactly P facilities are to be located. Constraint (8) links the location variables (X_j) and the allocation variables (Y_{ij}) . Constraints (9) and (10) are integrality constraints.

The solution to this model identifies the locations of the alert sites, the allocations of areas of interest to the alert sites, and the overall alert network distance. This model also assumes uncapacitated alert sites, single trips to each area of interest, separate trips to each candidate site and area of interest pair, and single site sourcing of demand. For fixed values of p the p-median problem can be solved in polynomial time; however, the problem is NP-hard for variable values of p (Garey and Johnson, 1979). Once the p-median formulation was conducted, the mathematical formulation of the p-center problem was conducted so that the worst case scenario could be determined.

Mathematical Formulation of the P-Center Problem

The objective of the p-center model is to minimize the maximum response time or distance between a supply site and a demand site. As discussed in Chapter 2, there are two different formulations of the p-center problem: the vertex p-center problem and the absolute p-center problem. The vertex p-center formulation will be used in this model because alert sites can only be located on the candidate alert site nodes and not on the arcs as in the absolute p-center problem. As in previous modeling techniques used in this

chapter, this modeling formulation will assume no limits on capacity at any candidate alert site. The original p-center problem was formulated by Hakimi (1964, 1965); however the formulation used in this research is from Daskin (1995) and Current et al. (2002). The formulation is as follows:

MINIMIZE W (11)

$$\sum_{j} Y_{ij} = 1 \qquad \forall i \tag{12}$$

$$\sum_{j} X_{j} = P \tag{13}$$

$$Y_{ij} \le X_j \qquad \forall i, j \tag{14}$$

$$W \ge \sum_{j} d_{ij} Y_{ij} \quad \forall i \tag{15}$$

$$X_j = 0,1 \qquad \forall j \tag{16}$$

$$Y_{ij} \ge 0 \qquad \forall i, j$$
 (17)

where

W = maximum distance between an area of interest and the nearest alert site

 $Y_{ij} = 1$ if area of interest i is assigned to alert site candidate j, 0 otherwise

 $X_j = 1$ if we locate at candidate alert site j, 0 otherwise

P = number of alert sites to locate; taken from LSCP results

 d_{ij} = distance from area of interest i to candidate alert site j

The objective function (11) minimizes the maximum distance that any area of interest is from an open alert site. Constraint (12) requires that each area of interest be assigned to exactly one alert site. Constraint (13) stipulates that P alert sites be located or

opened. Constraint (14) states that an area of interest *i* cannot be assigned to a candidate alert site *j* unless an alert site is located at *j*. Constraint (15) states that the maximum distance between an area of interest and an alert site must be greater than or equal to the distance between any area of interest *i* and the alert site *j* to which it is assigned. Constraints (16) and (17) are the respective integrality and non-negativity constraints. Once the aircraft operation assumptions were made, distance metric selection and distance calculations were accomplished so that the spreadsheet models could be constructed and solved.

Distance Metric Selection

As discussed in Chapter 2, there are four distinct distance metrics commonly used in location analysis. These metrics are: 1. Geographic distance (using latitude and longitude); 2. Euclidean or straight-line distance; 3. Routing factor; and 4. Rectilinear or Manhattan distance. Each of these metrics was explained in Chapter 2. Since the network involves aircraft flight from an alert site to an area of interest, the use of rectilinear distance metrics is eliminated. Also, given the fact that no airspace restrictions are assumed, routing factor distances are not used. The geographic metric was selected over the Euclidean distance metric because of the availability of the latitude and longitude coordinates and the desire for accuracy of the computed latitude-longitude distances. Specifically, the geographic metric takes into account the curvature of the earth using great circles while the Euclidean distance metric is exclusively a straight-line measure. The latitude-longitude coordinates for the areas of interest were obtained from the First Air Force AOC and the coordinates for the candidate alert sites were obtained

from the *DoD Flight Information Publication (Enroute)* dated 10 July 2003 to 4 September 2003. The actual coordinates are not presented in this research due to the sensitivity of the data. Once the coordinates were obtained, then, distances between areas of interest and candidate alert sites were computed.

Calculation of Geographic Distances

The calculation of geographic distances between alert site candidates and areas of interest are done by using the Haversine method. The Haversine method allows the calculation of distances between two locations on the earth's surface, as recommended by Sinnott (1984). This method compensates for the curvature of the earth through the use of great circles. The equations for calculating distance using the Haversine Method as described by Bell and McMullen (2003) are:

$$Dist_{ij} = r * (2 \tan^{-1}(\sqrt{b}, \sqrt{1-b}))$$
 (18)

$$b = \left[\sin\left(\frac{(\varphi_{i} - \varphi_{i})}{2}\right)\right]^{2} + \cos(\varphi_{i}) * \cos(\varphi_{i}) * \left[\sin\left(\frac{(\gamma_{i} - \gamma_{i})}{2}\right)\right]^{2}$$
 (19)

where

 $Dist_{ij}$ = Distance between area of interest *i* and candidate alert site *j*

r = Radius of the earth, equal to approximately 3437.67 nautical miles

 φ = Latitude of a candidate alert site or area of interest

 γ = Longitude of a candidate alert site or area of interest

The radius of the earth is entered in nautical miles to keep the units of measure consistent with best case aircraft flight time of 9 nautical miles per minute. Distance calculations were accomplished for each possible candidate alert site and area of interest combination

through the use of a C++ code developed by Bell and McMullen (2003). The code is included in Appendix A.

In order to use the code, the degree-minute-second latitude and longitude coordinates for each candidate site and each area of interest had to be converted to decimal form and entered into four different Microsoft Notepad text files. This conversion was accomplished by using the following widely known formula:

$$DD = d + \frac{m}{60} + \frac{s}{3600} \tag{20}$$

where

DD = Decimal degrees

d = Degrees

m = Minutes

s = Seconds

Neither the decimal computations nor the text files are included in this research because of the sensitivity of the information. The results of the calculations can be obtained from the author with the permission of the ACC Department of Homeland Security and the First Air Force AOC. The C++ code produces an output of a 202 X 70 distance matrix in an Excel® spreadsheet. The distance matrix is considered proprietary and not included in this research. The distances correspond to the nautical mile distances between each candidate alert site and each area of interest. After the distances were computed between the different nodes of the network, the critical distance for the LSCP model had to be computed.

Calculation of the Critical Distance

The calculation of the critical distance drives the computation of the objective function in the LSCP. The critical distance for use in the LSCP was calculated by the following formula:

$$S_{ij} = (MDRT - ACLT) * AS$$
 (21)

where

 S_{ij} = Critical distance from area of interest i to candidate alert site j

MDRT = Maximum Desired Response Time (From Table 3.1)

ACLT = Aircraft Launch Time

AS = Aircraft Speed (nautical miles per minute)

Essentially, S_{ij} is the dependent variable of the model. The value of S_{ij} is dependent on the values of the independent variables ACLT and AS. MDRT is a constant that changes based on the data in Table 3.1. Once the geographic and critical distances are calculated, then the spreadsheet models are constructed. Once the models are run with the computed critical distances, network response times for each critical distance can be computed

Calculation of Response Time

The calculation of the alert site response time is performed by utilizing the p-median solution for a given aircraft launch time and aircraft speed combination. The average network alert site response time is computed as follows:

$$ASRT_{ij} = \frac{p - median}{\frac{\#areas \, cov \, ered}{AS}} + ACLT$$
 (22)

where

 $ASRT_{ij}$ = Average site response time from alert site j to area i p-median = P-median solution given any computed critical distance in nautical miles

areas covered = Number of areas of interest covered in any particular model run ACLT = Aircraft Launch Time (in minutes)

AS = Aircraft Speed (nautical miles per minute)

The average alert site response time gives the average time that any alert site within a computed network configuration can respond to a covered area of interest.

Construction of the Spreadsheet Models

The choice to use a spreadsheet solver package was an easy one. This is because the LSCP and the p-center problems have respective one and three decision variables in their formulations. This makes them ill-suited for level curves or corner point enumeration. These methods were discussed in Chapter 2. A spreadsheet solver package is the best choice for all methods because it is capable of handling all three problem formulations. Since the decision was made to use a spreadsheet solver package, it was decided to use the Solver™ Add-In for the Microsoft Excel® spreadsheet package. This selection was driven by the author's personal preference and familiarity with the program. Due to the size of the model, the student version of the Premium Solver Platform™ was unable to be utilized in this research. A commercial version of the Premium Solver Platform™ as well as a Large Scale Linear Program Add-In were obtained from Frontline Systems, Incorporated. After deciding to use integer

programming and spreadsheet modeling, the different types of spreadsheet models had to be determined.

The location modeling mathematical formulations used in this research require the construction of two primary spreadsheet models. One for the LSCP and p-center problem and another for the p-median problem. The LSCP and p-center problem require one model because the maximum allowable distance constraint can be iteratively tightened on the LSCP, thus, minimizing the maximum distance between any area of interest and a candidate alert site. "Specifically, when the set covering problem equals p, the minimum associated coverage distance is the solution to the p-center problem" (Current et al., 2002:89). To ensure that the solution of each spreadsheet model is a global optimum solution as opposed to a local optima, the integrality constraints will be relaxed and the models re-run. The relaxed LP should produce the same solution as the ILP formulation. When a relaxed LP produces the same integer solution as the ILP formulation, a matrix is said to be totally unimodular. Since all of the proposed networks used in these studies are bipartite, then, the LP relaxed solution should equal the ILP solution, because bipartite graphs have been proven to unimodular. This concept was explained in Chapter 2. All model sets presented in this section are derivatives of the two primary models. Each model starts with the 202 X 70 distance matrix, which is produced by the C++ code. The two basic models are used to develop four different model derivatives.

Model Set I

Model Set I finds the LSCP, p-median, and p-center solutions for all Type I areas of interest. This model set produces the optimum permanent strip alert network given the previously stated objectives. This model will be run eight times varying the critical

distance on each run by adjusting launch times between 5-8 minutes in one minute increments and adjusting aircraft flight speed between 8 and 9 nautical miles per minute in one nautical mile per minute increments. The aircraft launch times and aircraft speeds are adjusted to see how sensitive solutions are to changes in S_{ij} . Candidate alert site number 69 retains its notional 5 minute launch time throughout this model set throughout the series of runs.

Model Set II

Model Set II finds the LSCP, p-median, and p-center solutions for all Type I and Type II areas. This model set produces the optimal locations for the permanent as well as flexible strip alert sites. The purpose of this model is to identify the optimal locations for the temporary Type II alert sites. Furthermore, this particular model seeks to identify any Type II sites that could also serve Type I areas of interest. Since the Type II areas of interest often vary, this model set is run once with the critical distance produced from the notional aircraft launch times and an aircraft speed of 9 nautical miles per minute.

Candidate alert site number 69 retains its notional 5 minute launch time.

Model Set III

Model Set III finds the LSCP, p-median, and p-center solutions for all Type I and Type II sites when 5 non-binding sites that are in the solution set from Model Set II are closed. This model set is used to show the model's utility at handling the closing of non-binding sites. The ability to handle runway closure is central to the effective implementation of the BRAC process. This model set is run once with an aircraft flight speed of 9 nautical miles per minute and notional launch times. Candidate alert site

number 69 retains its notional 5 minute launch time. The critical distance is computed from the mentioned parameters.

Model Set IV

Model Set IV finds the LSCP, p-median, and p-center solutions for all Type I areas of interest when only Air Force only candidate alert sites are considered. In this research, an Air Force only candidate alert site is a site used by the ANG, Air Force Reserve, or the active duty Air Force unit meeting the length of runway requirement. All other joint use strip alert candidates will be closed. This model will be run eight times varying the critical distance on each run by adjusting launch times between 5-8 minutes in one minute increments and adjusting aircraft flight speed between 8 and 9 nautical miles per minute in one nautical mile per minute increments. As in Model Set I, aircraft launch times and aircraft speed are adjusted to see how sensitive solutions are to changes in S_{ij} . Candidate alert site number 69 retains its notional 5 minute launch time throughout this model set.

Sensitivity Analysis

Sensitivity Analysis is conducted on all model sets. Model Sets I and IV are compared by examining the relationship between critical distance and the number of alert sites as well as the relationship between critical distance and aggregate network distance. The relationship between number of alert sites and aggregate network distance is discussed. Model Sets I and IV are compared based on average alert site response time. Additionally, common and binding alert sites for all Model Set I and IV runs are identified. This demonstrates sites that are insensitive to changes in the independent

variable values of launch time and aircraft within the ranges used in this research.

Finally, sensitivity of how the percent demand covered for each network configuration varies by removing n sites is explored for all model sets.

Summary

This chapter discussed the methodology for the analysis done in this research effort. Location analysis methods and solution techniques used in this study and their relevance were presented. First, the sources of data and methods of retrieval were introduced. Then, network objectives and critical model parameters were discussed, followed by network operation assumptions, and location modeling method selection and presentation. Additionally, solution technique selection was presented to show how the research question of optimal strip alert site location in the CONUS will be answered. Additionally, cursory steps in the spreadsheet modeling formulation such as distance metric selection and critical distance computation were presented to build the foundation for the spreadsheet models. Finally, the models constructed for analysis were presented to lay the foundation for the results and sensitivity analysis in Chapter 4.

IV. Results and Analysis

Introduction

This chapter summarizes the results of the model sets formulated in Chapter 3. Model Sets I and IV are run by varying the dependent variable of critical distance through the manipulation of the independent variables of aircraft launch time and aircraft speed. Model Sets II and III are run once with the notional launch times and maximum aircraft speed. Each location set covering problem (LSCP) model is run a second time with the integrality constraint relaxed to ensure the global optimality of every solution. All mileages shown are in nautical miles. A table and explanation of each set of results is presented to compile the solutions, and each candidate site and area of interest selected is identified by its specific number. After presenting the results of all model runs, the results of sensitivity analysis are shown to demonstrate how responsive the results of Model Sets I and IV are to changes in the independent variable values. Finally, the chapter ends by revisiting the research questions.

Model Set I

As discussed in Chapter 3, Model Set I considers all joint use airfields to cover the 66 Type I areas of interest. Areas of interest 28, 29, 30, and 70 are not considered in this model set because of their non-continuous nature of demand. The model set is run eight different times varying the critical distance (S_{ij}) through the manipulation of the aircraft launch times between 5-8 minutes in one minute increments, and aircraft speed between 8-9 NM per minute in one minute increments. Candidate site 69 keeps its ability

to launch its assigned aircraft within 5 minutes throughout all of the model runs. This allows candidate site 69 to cover any area of interest within a respective computed critical distances of 135 NM and 120 NM for all 9 NM and 8 NM per minute aircraft speed Model Set I runs. The rest of the candidate sites' critical distances are varied as indicated.

Notional (8-minute) Launch and 9 NM per minute Aircraft Speed Model

This set-up produces a critical distance of 108 NM for all Type I areas except for area 13, which has a critical distance of 36 NM. The difference is explained in Table 3.1 of Chapter 3. The initial run of the LSCP results in Solver™ being unable to find a feasible solution. Further investigation reveals that two areas of interest (38 and 66) do not have a candidate facility within the critical distance. The closest facility to area 38 is 141.753 NM and the closest facility to area 66 is 125.86 NM. This produces the p-center solution to this problem because the minimized maximum distance of the model is 141.753 NM. In order to run the model, both constraints were relaxed to 142 NM and 126 NM respectively. Another way of dealing with this problem is to establish continuous combat air patrols (CAPs) at both locations. This eliminates the need for strip alert coverage, but increases costs and resource consumption. In this research, areas that do not fall within the computed critical distance are covered with the relaxed critical distance as opposed to a continuous CAP.

Once the LSCP is run to produce the minimum number of sites to cover the demand, the p-median problem is solved to minimize aggregate network distance and to perform area of interest allocation to the respective sites. This is necessary in order to select from the many feasible solutions of the LSCP. Although snapshots of the actual

Solver™ results are included in Appendix B, Table 4.1 summarizes the model results for this network configuration.

Table 4.1. Results for Notional Launch/9 NM per minute Aircraft Speed Model

Table 4.1. Results for Notional Launch/9 NM per minute Aircraft Speed Model		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 31 alert sites	66 Type I areas covered	108 NM (areas 1-12,
p-center = 141.753 NM	w/31 alert sites	14-27, 31-37, 39-65; and
p-median = $3,151.115 \text{ NM}$		67-69); 36 NM (area 13);
Avg. dist./p-median = 47.744 NM		142 NM (area 38);
		and 126 NM (area 66)
Alert Site	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
6	13, 31	2
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
38	9, 68	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
69	23, 56	2
73	4, 40, 43	3
83	24	1
92	63	1
96	22, 54	2
104	39, 58	2
105	34	1
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
131	16, 55, 59	3
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
159	1, 32	2
170	17	1
189	61	1
195	14, 15, 51, 52, 53, 60	6

The solution to this model run also shows 9 binding alert sites. A binding alert site is one that must be part of the solution set because it is the only location that can cover a particular area of interest. Table 4.2 lists the binding alert sites for this solution and the areas of interest that require them to be in the solution set.

Table 4.2. Binding Sites for Notional Launch/9 NM per min. Aircraft Speed Model

Binding Sites (9)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
6	13
111	20, 21
23	33
105	34
118	38
73	40
92	63
49	66

As shown in Tables 4.1 and 4.2, the model produces a minimum solution of 31 joint use alert sites to cover the required 66 Type I areas of interest when run with notional aircraft launch times and 9 NM per minute aircraft speed. The minimum aggregate network distance produced from the 31 locations is 3,151.115 NM, while the minimized maximum distance or p-center solution is 141.753 NM. The average distance from site to area is computed by dividing the p-median network distance by the number of areas covered. Relaxing the integrality constraint and re-running the LSCP model as an LP produces the same results; therefore, this solution is optimal. Once the results were gathered for the model run with notional launch times and 9 NM per minute aircraft speed, the model was run with a one-minute reduction in launch time.

7-Minute Launch and 9 NM per minute Aircraft Speed Model

The computed critical distance for this input combination is 117 NM for all regular Type I areas of interest. Area 13's critical distance is increased to 45 NM with

the 1 minute reduction in aircraft launch. As is the case with the previous model, the distance constraints on areas 38 and 66 must be relaxed to 142 NM and 126 NM respectively in order for the model to run.

The solution to this critical distance combination is identical to the previous model run. The snapshots of the actual Solver™ results are included in Appendix B. As shown, the minimum number of alert sites is 31, while the objective function values for the p-median and p-center solutions are 3,151.115 NM and 141.753 NM respectively. The alert site locations and areas covered are identical. As in the last model, a minimum of 31 alert sites are needed to cover the 66 Type I areas of interest. Additionally, a re-run of the LSCP model relaxing the integrality constraint produces the same solution. Therefore, the solution is again optimal. The binding locations are the same as the previous model with one exception. Location 73 is excluded as a binding location. Therefore, the number of binding locations in this model is 8 as opposed to 9 in the previous model. After determining the 7-minute launch solution, the launch time was reduced again by one minute and the model re-run.

6-Minute Launch and 9 NM per minute Aircraft Speed Model

The regular Type I site critical distance produced from the parameters of a 6-minute launch and 9 NM per minute aircraft speed is 126 NM. The critical distance for area 13 is increased to 54 NM for this model. As is the case in previous models, the distance constraint on area of interest 38 must be increased to 142 NM in order for the model to run. Unlike previous models, area 66 falls within the regular Type I area critical distance for this model, therefore, it is not necessary to increase its critical distance.

Snapshots of the actual Solver™ results are included in Appendix B. The solutions generated from this particular critical distance are included in Table 4.3.

Table 4.3. Results for 6-Minute Launch/9 NM per minute Aircraft Speed Model

1 able 4.5. Results for 6-Minute Launch/9 NM per minute Aircraft Speed Model		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 28 alert sites	66 Type I areas covered	126 NM (areas 1-12,
p-center = 141.753 NM	w/28 alert sites	14-27, 31-37, and 39-69);
p-median = $3,466.072 \text{ NM}$		54 NM (area 13); and
Avg. dist./p-median =		142 NM (area 38)
52.516 NM		
Alert Site	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
18	15, 16, 51, 55, 59	5
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
38	9, 68	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
59	6, 24, 69	3
69	23, 56	2
73	4, 40, 43	3
104	39, 58	2
105	34, 63	2
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
137	13, 14, 31, 52, 53, 60	6
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
159	1, 32	2
170	17	1
189	61	1
201	22, 54	2
	,	

With an increase in critical distance to 126 NM, the LSCP portion of this model shows that a minimum number of 28 alert sites are capable of covering all 66 Type I

areas of interest. This is a reduction of three required alert sites from the previous two models. Subsequently, fewer sites required to cover all areas causes the p-median aggregate network distance solution to increase to 3,466.072 NM. Also, different alert sites are selected than in the previous two models because the increased critical distance permits the coverage of a wider area from each site. The objective function value for the p-center solution remains unchanged because minimized maximum distance is 141.753 NM as in the previous two models. Not only does the increase in critical distance cause a reduction in the number of required alert sites, it also reduces the number of binding alert sites to 3. The binding sites and the areas of interest causing the binding conditions are shown in Table 4.4.

Table 4.4. Binding Sites for 6-Min. Launch/9 NM per minute Aircraft Speed Model

Binding Sites (3)	Area(s) Causing Binding Condition
47	8, 50
118	38
49	66

As is the case of the previous models, relaxing the integrality constraint in this model and solving as an LP had no affect on the optimal LSCP solution found by SolverTM. The minimum number of sites remains at 28. Therefore, 28 sites is an optimal solution at the computed critical distance. Once optimality is verified, the aircraft launch time is reduced by one minute and the model is run again.

5-Minute Launch and 9 NM per minute Aircraft Speed Model

The computed critical distance for regular Type I areas using 5-minute launch and 9 NM per minute flight time is 135 NM. Also, the critical distance for area 13 is increased to 63 NM from previous models. As in previous models, the critical distance for area 38 is increased to 142 NM in order for the model to find a solution. The actual

Solver[™] results from the model runs utilizing the aforementioned parameters are included in Appendix B. Model results are compiled and presented in Table 4.5.

Table 4.5. Results for 5-Minute Launch/9 NM per minute Aircraft Speed Model

Results	Coverage Scheme	Critical Distance (S_{ii})
$\overline{LSCP} = 26$ alert sites	66 Type I areas covered	135 NM (areas 1-12,
p-center = 141.753 NM	w/26 alert sites	14-27, 31-37, and 39-69);
p-median = 3,954.265 NM		63 NM (area 13); and
Avg. dist./p-median =		142 NM (area 38)
59.913 NM		
Alert Site	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
18	15, 16, 51, 55, 59	5
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
59	6, 24, 61, 69	4
69	23, 56	2
73	4, 40, 43	3
105	34, 63	2
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
137	13, 14, 31, 52, 53, 60	6
138	18	1
143	12, 36, 57, 58	4
151	9, 39, 68	3
152	25, 41	2
156	27, 65	2
159	1, 32	2
170	17	1
201	22, 54	2

As seen in Table 4.5, increasing the critical distance to 135 NM results in a 2 site reduction of the minimum number sites to cover all the demand from the previous model. The LSCP solution for this particular model is an optimal 26 alert sites. Optimality is

verified using the same technique as previous models. Also, the reduction to 26 alert sites causes an increase in minimized aggregate network distance to 3,954.265 NM for the p-median solution. The objective function value for the p-center solution (141.753 NM) for this model is unchanged from previous model configurations; however, it is now a 26-center solution.

The increase in aggregate network distance in this model corresponds to the way the previous models have reacted to a reduction in the minimum number of alert sites. Not only does the increase in critical distance produce a decrease in the minimum required number of alert sites, it also results in a reduction in the number of binding locations for the model. The binding alert sites for the 5 minute launch and 9 NM per minute flight time model are listed in Table 4.6.

Table 4.6. Binding Sites for 5-Min. Launch/9 NM per minute Aircraft Speed Model

Binding Sites (2)	Area(s) Causing Binding Condition
47	8
118	38

The number of binding alert sites is reduced from 3 to 2 from the previous model. This reduction is produced from the increase in critical distance, because the increased critical distance permits more coverage options. After Model Set I is varied by launch time at 9 NM per minute aircraft speed, the model set is adjusted by launch time at 8 NM per minute aircraft flight speed.

Notional (8-minute) Launch and 8 NM per minute Aircraft Speed Model

These launch and flight speed parameters produce a critical distance of 96 NM for all regular Type I areas of interest. Area 13's critical distance is reduced to 32 NM for this particular model. Also, although candidate site 69 keeps its ability to launch its

aircraft in 5 minutes, the reduction in aircraft flight speed produces a critical distance of 120 NM as opposed to the 135 NM critical distance used in the previous models. As previously discussed, candidate site 69 will keep this critical distance throughout all of the 8 NM per minute flight speed models. Also, the reduction in critical distance to 96 NM increases the number of areas where the critical distance constraints must be relaxed. As seen in all of the previous models, the distance constraint for area 38 must be relaxed to 142 NM. Also, the distance constraint for area 66 must be relaxed to 126 NM. In addition to these areas, areas 33 and 37 cannot be met with regular distance constraints. The closest candidate alert site to each are located at 102.4721 NM to area 33 and 98.2689 NM to area 37. Therefore, the distance constraints on areas 33 and 37 are relaxed to 103 NM and 99 NM respectively to allow the model to run. Although snapshots of the actual Solver™ results are included in Appendix B, Table 4.7 summarizes the model run results for this network configuration.

Table 4.7. Results for Notional Launch/8 NM per minute Aircraft Speed Model

	tional Launch/8 NM per min	
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 32 alert sites	66 Type I areas covered	96 NM (areas 1-12,
p-median = 3,097.354 NM	w/32 alert sites	14-27, 31-32, 34-36, 39-65,
p-center = 141.753 NM		and 67-69); 32 NM (area
Avg. dist./p-median = 42.93		13); 103 NM (area 33); 99
NM		NM (area 37); 142 NM
		(area 38); and 126 NM
		(area 66)
Alert Site	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
6	13, 31	2
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
36	24, 69	2
38	9, 68	2
46	3, 44	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
57	6	1
69	23, 56	2
73	4, 40, 43	3
92	63	1
96	22, 54	2
104	39, 58	2
105	34	1
111	20, 21	2
118	38	1
121	26, 46, 48	3
131	16, 55, 59	3
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
159	1, 32	2
170	17	1
187	2	1
189	61	1
195	14, 15, 51, 52, 53, 60	6
1/3	17, 13, 31, 34, 33, 00	U

The solution to the LSCP model with the 1 NM reduction in aircraft speed and notional launch times increases to a minimum of 32 required joint use alert sites. There are 7 other different launch time/aircraft speed configurations. This increase can be explained by the reduction in critical distance. All models are demonstrating an inverse relationship between critical distance and the number of alert sites. Also, the models show an inverse relationship between the required number of alert sites and the aggregate network distance of the p-median solution and a direct relationship between critical distance and the aggregate network distance of the p-median solution. Specifically, this model shows that the increase in the minimum number of alert sites to 32 causes a reduction in the p-median aggregate network distance to 3,097.354 NM. The required minimum of 32 alert sites is proven to be an optimal solution through LP relaxation as is the case in previous models. Not only does the reduction in critical distance cause an increase in the minimum number of required alert sites to cover all demand, it also produces an increase in the number of binding alert sites.

The binding alert sites and the areas causing the binding conditions are shown in Table 4.8. The table shows an increase in the number of binding sites to 10.

Table 4.8. Binding Sites for Notional Launch/8 NM per min. Aircraft Speed Model

Binding Sites (10)	Area(s) Causing Binding Condition
57	6
47	7, 8, 49, 50
6	13
111	20, 21
23	33
105	34
49	37, 66
118	38
73	40
92	63

Table 4.8 also shows an increase in the number of areas causing the binding condition. Hence, a smaller critical distance produces less coverage options in the overall network. Therefore, a larger number of binding sites must be in the solution set. Once the model solution is computed at 8 NM per minute aircraft speed with notional launch times, the launch time was decreased by one minute as is done in all the 9 NM aircraft speed models.

7-Minute Launch and 8 NM per minute Aircraft Speed Model

The one minute reduction in launch time from 8 minutes to 7 minutes at an aircraft speed of 8 NM per minute produces a computed critical distance of 104 NM, with a computed critical distance for area 13 of 40 NM. With a computed critical distance of 104 NM, the critical distances on areas 38 and 66 must be relaxed to respective 142 NM and 126 NM as is done in many of the previous models to allow the model to run. Although the computed critical distance for this model is unique, the solution is not.

The optimal solution for this particular launch and aircraft speed combination results in a LSCP, p-median, p-center, and coverage allocation solution identical to the notional launch/9 NM per minute aircraft speed model shown earlier in this set. To restate the solutions, the LSCP solution requires a minimum of 31 alert sites; the p-median solution shows a minimum aggregate network distance of 3,151.115 NM; and the p-center solution is 141.753 NM. Furthermore, the number of binding alert sites is 9, which correspond to the same number and location of the binding sites in the notional launch/9 NM per minute model. Finally, the number and location of the areas causing the binding conditions are identical to the previously mentioned model. Although the snapshots of the actual Solver™ results are included in Appendix B, the reader is referred

to Tables 4.1 and 4.2 earlier in this section for a compilation of the results. After the 7-minute launch and 8 NM per minute aircraft speed model is run, the launch time is reduced again by one minute to re-run the model and gather the results.

6-Minute Launch and 8 NM per minute Aircraft Speed Model

An aircraft launch time of 6 minutes, coupled with an 8 NM per minute aircraft speed, produces a computed network critical distance of 112 NM. The computed critical distance for the coverage of area 13 is increased to 48 NM. As in previous models, the critical coverage distances of areas 38 and 66 have to be relaxed to respective 142 NM and 126 NM to allow the model to run. The actual Solver™ results of the LSCP and p-median models are included in Appendix B. As in the previous model, the optimal solutions for the LSCP, p-median, and p-center algorithms as well as the specific alert sites and area allocation in this model are the same as the notional launch/9 NM per minute aircraft speed model. The globally optimal minimum LSCP solution is 31 alert sites. The minimum total network distance or the p-median solution is 3,151.115 NM. Finally, the p-center solution, or the minimized maximum distance, is again 141.753 NM. For the compiled results of the models, the reader is referred to Table 4.1. While the location results of this model mirror the results of the notional launch/9 NM per minute aircraft speed model, this model differs in the number of binding sites.

There are 8 binding alert sites in this particular model. This is one less site than the two models that this one emulates in coverage scheme and location solution. The binding locations and areas causing the binding conditions for this model are shown in Table 4.9.

Table 4.9. Binding Sites for 6-Min. Launch/8 NM per minute Aircraft Speed Model

Binding Sites (8)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
111	20, 21
23	33
105	34
118	38
73	40
92	63
49	66

The reduction in binding sites is attributed to the increase in critical distance from the notional launch/9 NM per minute aircraft speed and the 7-minute launch/8 NM per minute aircraft speed models. Those models produce respective critical distances of 108 NM and 104 NM as compared to the 112 NM computed critical coverage distance of this model. Although this model shows very little change from its immediate predecessor, the 6-minute launch/8 NM per minute model produces changes that are more pronounced.

5-Minute Launch and 8 NM per minute Aircraft Speed Model

The computed critical distance for the 5-minute launch/8 NM per minute aircraft speed model produces a critical distance of 120 NM. The computed critical distance for area 13 with the independent variable values for this model is 56 NM. Critical distances for areas 38 and 66 are relaxed in this model to respective values of 142 NM and 126 NM as they are in several of the previous models. The p-center solution for this model is 141.753 NM, which corresponds to the closest candidate site to area 38. Although the p-center solution for this model is the same as all of the other models, the LSCP and p-median solutions differ from previous solutions. The actual results for the LSCP and p-median models are presented in Appendix B. Synthesized results for the LSCP, p-median, and p-center models are shown in Table 4.10.

Table 4.10. Results for 5-Minute Launch/8 NM per minute Aircraft Speed Model

Table 4.10. Results for 5-Minute Launch/8 NM per minute Aircraft Speed Model		
Results	<u>Coverage Scheme</u>	Critical Distance (S_{ij})
LSCP = 29 alert sites	66 Type I areas covered	120 NM (areas 1-12,
p-median = $3,318.896$ NM	w/29 alert sites	14-27, 31-32, 34-37, 39-65,
p-center = 141.753 NM		and 67-69); 56 NM (area
Avg. dist./p-median =		13); 142 NM (area 38); and
50.286 NM		126 NM (area 66)
<u>Alert Site</u>	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
18	15, 16, 51, 55, 59	5
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
38	9, 68	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
69	23, 56	2
73	4, 40, 43	3
83	24	1
96	22, 54	2
104	39, 58	2
105	34, 63	2
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
137	13, 14, 31, 52, 53, 60	6
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
159	1, 32	2
170	17	1
189	61	1
107	V 1	*

As seen in Table 4.10 the minimum number of locations required to cover all 66 Type I areas of interest with a critical distance of 120 NM is 29. Re-solving the LSCP problem using LP relaxation proves this solution to be globally optimal. The number of

sites follows the established inverse relationship between critical distance and minimum number sites. With the reduction in sites, the p-median solution increases to a total network distance of 3,318.896 NM. This solution has a direct relationship with critical distance and an inverse relationship with the number of sites. Not only is the number of joint use alert sites decreased over previous 8 NM aircraft speed models, but so is the number of binding sites. The 5-minute launch/8 NM per minute aircraft speed model produces 4 binding alert sites in its solution set. These sites and the areas causing the binding conditions are shown in Table 4.11.

Table 4.11. Binding Sites for 5-Min. Launch/8 NM per min. Aircraft Speed Model

Binding Sites (4)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
111	20
118	38
49	66

Many of the binding alert sites in this model are common to other models. Although all previous models consider the 66 Type I areas of interest, they do not take into account the 4 Type II areas. The next section presents the results of Model Set II, which looks at optimizing Type I and Type II areas.

Model Set II

Model Set II finds the LSCP, p-median, and p-center solutions for all Type I and Type II areas of interest combined from the 202 joint use candidate alert sites. The overarching objective of this model is to find the best locations for the Type II areas of interest. Type II areas in this model are treated as permanent areas for ease of modeling. Since the Type II areas of interest often vary, this model set is run once with notional launches and 9 NM per minute aircraft speed. Candidate alert site 69 retains its ability to

launch its assigned aircraft in 5 minutes for this model. This corresponds to a computed critical distance of 135 NM. Furthermore, the model set identifies alert sites that can serve Type II as well as Type I areas of interest given the independent variable values. Although covering both types of areas with the same alert site prevents ACC from having to deploy aircraft, equipment, and personnel to an alert site to cover only a Type II area alone, the non-repetitive demand experienced in the Type II areas makes sacrificing overall network performance for the Type I areas undesirable.. As discussed in Chapter 3, the Type II areas of interest are areas 28, 29, 30, and 70. These areas require alert aircraft on a non-continuous basis when requested by NORAD.

The results from this model run are presented in the same format as the Model Set I results. The computed critical distance for this particular model is 108 NM given the notional (8-minute) launch and 9 NM aircraft flight speed. Area 13's computed critical distance for this configuration is 36 NM. Also, the distance constraints on areas 38 and 66 must be increased to respective 142 NM and 126 NM to facilitate the running of the model due to lack of viable candidate sites within the computed critical distance. In addition to areas 38 and 66, the closest candidate site to area 70 is 163.8086 NM, therefore, the critical distance for area 70 must be relaxed to 164 NM to allow the model to run. Ultimately, the objective function value for the p-center solution for this model is 163.8086 NM, because the distance to area 70 represents the minimization of the maximum distance in the network. The results of the LSCP and p-median models are presented in Table 4.12. The actual Solver™ results are included in Appendix C.

Table 4.12. Results for Model Set II

Table 4.12. Results for Model Set II		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 33 alert sites	70 total areas66 Type I	108 NM (areas 1-12,
p-center = 163.8086 NM	and 4 Type II areas covered	14-27, 31-37, 39-65, and
p-median = 3,512.511 NM	w/33 alert sites	67-69); 56 NM (areas 13,
Avg. dist./p-median =	32 permanent sites	28-30); 142 NM (area 38);
50.179 NM	1 non-permanent site	126 NM (area 66); and 164
		NM (area 70)
<u>Alert Site</u>	Area(s) Covered	# Areas Covered
1	64	1
2	45, 62	2
6	13, 14, 31	3
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
38	9, 68	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
67	27, 65, 70	3
69	23, 56	2
73	4, 40, 43	3
83	24	1
92	63	1
96	22, 54	2
98	30, 53	2
104	39, 58	2
105	34	1
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
135	16, 28, 59	3
138	18	1
148	12, 36, 57	3
150	29	1
152	25, 41	2
159	1, 32	2
170	17	1
174	15, 51, 52, 55, 60	5
189	61	1
107	U1	1

The solution to the LSCP shows that a minimum of 33 sites can cover all Type I and Type II areas of interest. This is proven optimal through LP relaxation. The p-median solution gives an aggregate network distance of 3,512.511 NM. As can be seen in Table 4.12, Type II areas 28, 30, and 70 are covered by alert sites that also cover Type I areas of interest. However, none of the alert sites in the model that cover Type II areas are optimal locations for any of the Model Set I configurations. This is explained by examining the binding locations for the model.

The binding locations for this model set are shown in Table 4.13. The table shows 13 binding locations; 9 for Type I areas and 4 for the Type II areas.

Table 4.13. Binding Alert Sites for Model Set II

D: 1: 0: (10)	A () G : D: I: G I::	A E ()
Binding Sites (13)	Area(s) Causing Binding Condition	Area Type(s)
47	7, 8, 49, 50	Type I
6	13	Type I
111	20, 21	Type I
23	33	Type I
105	34	Type I
118	38	Type I
73	40	Type I
92	63	Type I
49	66	Type I
135	28	Type II
150	29	Type II
98	30	Type II
67	70	Type II

A closer look at the binding Type II alert sites shows that all sites covering Type II areas are binding; therefore, each Type II area can be served by only one candidate alert site given the input parameters. Although three of the four Type II alert sites can also cover Type I areas of interest as shown in Table 4.12, making the Type II alert sites permanent locations, as opposed to temporary, would degrade the performance of the permanent alert network. This occurs because the binding conditions on all Type II alert sites force

the model to keep these sites open. Specifically, each Type II area can only be served by one specific candidate site, therefore, the site must be selected. Finally, since each of the Type II areas are served by one site, the optimal alert site locations for areas 28, 29, 30, and 70 can be obtained from Tables 4.12 and 4.13. Not only does the model have utility at identifying optimal alert sites for Type II areas, it is also responsive to changes in the candidate alert sites.

Model Set III

Model Set III finds the LSCP, p-center, and p-median solutions after taking 5 non-binding sites out of the solution set of Model Set II. The purpose of taking the 5 sites out of the solution set is to show the model's ability to adapt to changes in the number of potential candidate sites. Ultimately, this demonstrates the model's usefulness at evaluating potential runway closure decisions. As in Model Set II, all Type I and Type II areas of interest require coverage. After closing or withdrawing 5 non-binding sites, the possible candidate site set is reduced to 197 sites instead of the 202 used in Model Set II. Taking a binding alert site out of the solution set would make finding a solution infeasible; therefore, only non-binding sites are withdrawn. The aircraft launch times and aircraft speed in this model set is identical to Model Set II. Consequently, the computed critical distances are also the same as Model Set II. Additionally, the critical distances for areas 38, 66, and 70 must be relaxed to the critical distances used in Model Set II in order for the model to successfully run. The candidate alert sites removed from this model set are 1, 2, 20, 24, and 38. These sites were randomly chosen because they were the first five non-binding sites in the Model Set II solution. As previously stated, each of

these alert sites is in the solution set for Model Set II. Once the alert sites are closed or removed from consideration, the model is solved.

The LSCP minimum number of sites and objective function of the p-center solution for this model do not change from Model Set II. The optimal minimum number of alert sites to cover all Type I and Type II areas is 33 sites. Area 70's closest candidate facility dictates the objective function value of the p-center solution for this model set as it did in Model Set II. The objective function value for the p-center solution in this model set is 163.8086 NM. The actual Solver™ results are included in Appendix D. The compiled results of the model run are shown in Table 4.14.

Table 4.14. Results for Model Set III

Parella Carres Calana Critical Distance (C)		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 33 alert sites	70 total areas66 Type I	108 NM (areas 1-12,
p-center = 163.8086	and 4 Type II areas covered	14-27, 31-37, 39-65, and
p-median = 3,722.806 NM	w/33 alert sites	67-69); 56 NM (areas 13,
Avg. dist./p-median =	32 permanent sites	28-30); 142 NM (area 38);
53.183 NM	1 non-permanent site	126 NM (area 66); and 164
		NM (area 70)
Alert Site	Area(s) Covered	# Areas Covered
6	13, 14, 31	3
23	33	1
31	5, 35, 42, 67	4
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
67	27, 65, 70	3
69	23, 56	2
73	4, 40, 43	3
83	24	1
86	9, 68	2
89	64	1
92	63	1
96	22, 54	2
98	30, 53	2
104	39, 58	2
105	34	1
111	20, 21	2
113	2, 3, 44	3
118	38	1
121	26, 46, 48	3
130	10, 11	2
135	16, 28, 59	3
138	18	1
148	12, 36, 57	3
150	29	1
152	25, 41	2
159	1, 32	2
161	45, 62	2
170	17	1
174		5
	15, 51, 52, 55, 60	
189	61	1

While the LSCP and p-center solutions are not affected by closing the 5 non-binding sites, this is not the case for the p-median solution or the number of binding locations. The objective function value of the p-median solution for this model is 3,722.806 NM. Closing the 5 previously mentioned sites causes an increase in aggregate network distance of 210.295 NM. Any closure will cause an increase, but the extent of the increase is dependent on which sites are selected for closure. Not only is the p-median solution increased with closure of 5 non-binding locations, but so is the number of binding locations.

Closing the 5 aforementioned sites in this model set causes a 3 site increase in the number of binding alert sites from Model Set II. This occurs because the number of candidate sites is reduced with closure, which forces some areas into a binding coverage condition. The list of binding sites for this model is shown in Table 4.15.

Table 4.15. Binding Alert Sites for Model Set III

Binding Sites (16)	Area(s) Causing Binding Condition	Area Type(s)
47	7, 8, 49, 50	Type I
6	13	Type I
111	20, 21	Type I
23	33	Type I
105	34	Type I
118	38	Type I
73	40	Type I
161	45, 62	Type I
92	63	Type I
89	64	Type I
49	66	Type I
31	67	Type I
135	28	Type II
150	29	Type II
98	30	Type II
67	70	Type II

As discussed in Model Set II, the Type II sites can only be covered by one site each given the input parameters, therefore, each Type II alert site is binding. Decisions to close bases can force coverage decisions by driving sites into binding conditions. For example, alert site 2 covers areas 45 and 62 in Model Set II and is non-binding. In Model Set III, site 2 is closed and areas 45 and 62 are covered by alert site 161 and are binding. These factors must be considered, and when evaluating sites for closure, effects on the entire network must be considered. Although what-if scenarios are explored in Model Sets II and III, Air Force only candidate sites are considered in the coverage of Type I areas in Model Set IV.

Model Set IV

Model Set IV finds the LSCP, p-median and p-center solutions for the coverage of all Type I areas of interest when Air Force only alert sites are considered. This involves closing all joint use candidate airfields that are not affiliated with the Air Force. As discussed in Chapter 3, an Air Force only candidate alert site is a site used by the ANG, Air Force Reserve, or the active duty Air Force. This model set is run with the same independent variable values as are used in Model Set I. Subsequently the computed critical values for this model set in each scenario equal the computed critical values for Model Set I. Also, as in Model Set I, candidate site 69 keeps it ability to launch its assigned aircraft within 5 minutes throughout all of the model runs. This allows candidate site 69 to cover any area of interest within respective computed critical distances of 135 NM and 120 NM for all Model Set IV runs with 9 NM and 8 NM per

minute aircraft speeds. The model categories for Model Set IV are the same as those used in Model Set I.

Notional (8-minute) Launch and 9 NM per minute Aircraft Speed Model

As in Model Set I, the computed critical distance for these independent variable values is 108 NM. Similarly, area 13's critical distance is 36 NM for this model. Also, areas 38 and 66 require relaxation of the critical distance in order for the model to run. These areas require the increase in critical distances to respective 142 NM and 126 NM. Also, area 44 requires an increase of its coverage distance to 111 NM, because the closest candidate site to this location is 110.4306 NM. The Solver™ results of the Air Force only model runs utilizing these parameters are included in Appendix E. The compiled results of this model are shown in Table 4.16.

Table 4.16. Results for Notional Launch/9 NM per minute Aircraft Speed Model

Table 4.16. Results for Notional Launch/9 NM per minute Aircraft Speed Model		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 32 alert sites	66 Type I areas	108 NM (areas 1-12,
p-center = 141.753 NM	covered	14-27, 31-37, 39-65; and
p-median = 3,315.869 NM	w/32 alert sites	67-69); 36 NM (area 13);
Avg. dist./p-median = 50.24		142 NM (area 38); 111 NM
NM		(area 44); and 126 NM (area
		66)
Alert Site	Area(s) Covered	# Areas Covered
2	45, 62	2
6	13, 31	2
15	68	1
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
27	58	1
37	61	1
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
69	23, 56	2
73	4, 40, 43	3
83	24	1
89	64	1
92	63	1
96	22, 54	2
97	2, 3, 44	3
101	17	1
105	34	1
111	20, 21	2
115	1, 32	2
118	38	1
121	26, 46, 48	3
131	16, 55, 59	3
138	18	1
148	12, 36, 57	3
151	9, 39	2
152	25, 41	2
156	27, 65	2
195	14, 15, 51, 52, 53, 60	6
1/0	1 ., 10, 01, 02, 00, 00	V

The LSCP for this model produces a minimum of 32 required Air Force only sites to cover all Type I areas of interest. This solution is optimal and exceeds the comparable Model Set I run by 1 site. The objective function value of the p-center solution for this model is 141.753 NM, which corresponds to the Model Set I counterpart. This model produces a p-median solution of 3,315.869 NM, which exceeds the minimized aggregate network distance of the Model Set I solution by approximately 165 NM. The 1-site and aggregate network distance increases in this model from the Model Set I version can be explained by the reduction in candidate sites. Subsequently, limiting the candidate sites to Air Force only also causes the number of binding alert sites to increase from the Model Set I version.

The binding alert sites for the Air Force only version of the notional launch/9 NM per minute aircraft speed model are shown in Table 4.17. The 17 binding sites eclipse the comparable joint use model by 8 alert sites.

Table 4.17. Binding Sites for Notional Launch/9 NM per min. Aircraft Speed Model

Binding Sites (17)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
151	9
6	13
101	17
111	20, 21
69	23, 56
23	33
105	34
148	36
49	37, 66
118	38
73	40
97	44
2	45, 62
37	61
92	63
89	64

The dramatic increase in the number of binding alert sites can be attributed to the reduction of candidate sites, which forces many areas into binding coverage conditions. This concept is explored in Model Set III. After running the Air Force only model with notional launches and a 9 NM per minute aircraft speed, the launch time is reduced by 1 minute and the model re-run.

7-Minute Launch and 9 NM per minute Aircraft Speed Model

The Air Force only 7-minute launch and 9 NM per minute aircraft speed model has a computed critical distance of 117 NM, which corresponds to the value computed for Model Set I with the same parameters. The area 13 critical distance of 45 NM is also unchanged from the corresponding model in Model Set I. Finally, as in the Model Set I derivative of this particular model, the critical distances of areas 38 and 66 must be relaxed to respective 142 NM and 126 NM in order for the model to run. As in Model

Set I, the closest candidate sites to areas 38 and 66 are respective 141.753 NM and 125.86 NM. While the input parameters for this model are the same as its Model Set I counterpart, the results of this model are nearly identical to the previous Air Force only model.

The LSCP, p-median, and p-center objective function values for the input parameters given Air Force only candidate sites are the same as those found in the notional launch/9 NM per minute aircraft speed Air Force only model. The reader is referred to Table 4.16 for a compilation of the model results. The critical distances used to obtain the solutions in Table 4.16 are replaced with those from the previous paragraph. Every other result is identical in this model. The minimum of 32 required alert sites is optimal, which is verified through LP relaxation. The SolvertM results of the runs utilizing the 7-minute launch/9 NM per minute aircraft speed parameters are included in Appendix E. As is the case in the previous Air Force only model, the Model Set I solutions for the given input parameters requires one less minimum alert site (31) and reduces the p-median solution by approximately 165 NM. While the solutions of this particular model mirror the notional launch/9 NM per minute aircraft speed model answers and compare comparably to the Model Set I counterpart, there is a slight difference in the number of binding alert sites.

The number of binding sites in this model is 16. This solution is 1 less than the binding sites computed for the previous Air Force only model. The reader is referred to Table 4.17 for a breakdown of the binding alert sites and the areas causing the binding conditions. The only difference between the binding locations for this model and the previous model is that location 89 in this model is not binding. The 9 NM increase in

critical distance from the previous model produces one less binding alert site because of the larger coverage area. After gathering the results from the 7-minute launch/9 NM per minute aircraft speed Air Force only model, solutions are computed for the Air Force only derivative with a one minute reduction in aircraft launch.

6-Minute Launch and 9 NM per minute Aircraft Speed Model

The Air Force only 6-minute launch/9 NM per minute aircraft speed model has a computed critical distance of 126 NM. The critical distance for area 13 is 54 NM. Both values are the same as the values computed in the comparable model in Model Set I. With a computed critical distance of 126 NM, only the critical distance of area 38 requires relaxation to 142 NM in order for the model to run. Solver™ results of the Air Force only model runs for the mentioned parameters are included in Appendix E. The results of the model runs are compiled in Table 4.18.

Table 4.18 shows the results of the LSCP, p-median algorithm, and p-center problems. It also contains the computed critical distances and coverage scheme for this model.

Table 4.18. Results for 6-Minute Launch/9 NM per minute Aircraft Speed Model

Pagulta		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 29 alert sites	66 Type I areas covered	126 NM (areas 1-12,
p-center = 141.753 NM	w/29 alert sites	14-27, 31-37; and 39-69);
p-median = 3,533.369 NM		54 NM (area 13); and
Avg. dist./p-median = 53.536 NM		142 NM (area 38)
Alert Site	Area(s) Covered	# Areas Covered
2	45, 62	2
6	13, 31	2
15	9, 68	2
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
37	61	1
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
59	6, 24, 69	3
69	23, 56	2
73	4, 40, 43	3
89	64	1
97	2, 3, 44	3
101	17	1
104	39, 58	2
105	34, 63	2
111	20, 21	2
115	1, 32	2
118	38	1
121	26, 46, 48	3
135	16, 55, 59	3
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
195	14, 15, 51, 52, 53, 60	6
201	22, 54	2
4 √1	, J I	-

The LSCP result for the Air Force only version 6-minute launch/9 NM per minute aircraft speed model produces a global minimum of 29 sites to cover all 66 Type I areas interest.

This is one site more than is required to cover all of the areas of interest in Model Set I when joint sites are considered using the input parameters. The p-center objective

function value is 141.753 NM, which corresponds to the minimum distance to cover area 38. This solution is identical to Model Set I. The aggregate network distance from the p-median solution for this model is 3,533.369 NM. The p-median solution's aggregated network distance for this model is 67.297 NM greater than the p-median answer obtained in Model Set I, but Model Set I achieves its aggregate network distance with one less site. Not only does this model show a reduction in the minimum number of required alert sites over previous Air Force only models, it also shows a reduction in the number of binding alert sites.

The number of binding alert sites for the Air Force only 6-minute launch/9 NM per minute aircraft speed model is reduced to 7 locations from previous Air Force only candidate site models. The binding alert sites and the areas causing the binding conditions are shown in Table 4.19.

Table 4.19. Binding Sites for 6-Min. Launch/9 NM per min. Aircraft Speed Model

Binding Sites (7)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
69	23, 56
148	36
49	37, 66
118	38
97	44
2	45

Although the number of binding alert sites is reduced by 9 from the previous Air Force only model, the number of binding alert sites for this model is 4 more than its Model Set I counterpart. This is explained by the fact that Model Set I has more candidate sites, which means a greater number of coverage options.

5-Minute Launch and 9 NM per minute Aircraft Speed Model

Decreasing the aircraft launch time by one additional minute while holding the aircraft speed constant at 9 NM per minute produces a critical distance of 135 NM. The area 13 critical distance is increased to 63 NM. As is the case in all Model Set I and Model Set IV runs, the critical distance for area 38 must be extended to 142 NM in order for the model to run. A snapshot of the Solver™ results of the Air Force candidate site only model runs corresponding to the independent variable values in this model are included in Appendix E. The solutions to the model are compiled in Table 4.20.

Table 4.20. Results for 5-Minute Launch/9 NM per minute Aircraft Speed Model

Table 4.20. Results for 3-Minute Launen/7 NWI per influte Affer are speed Mode		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 27 alert sites	66 Type I areas covered	135 NM (areas 1-12,
p-center = 141.753 NM	w/27 alert sites	14-27, 31-37; and 39-69);
p-median = $3,740.625 \text{ NM}$		63 NM (area 13); and
Avg. dist./p-median= 52.58 NM		142 NM (area 38)
Alert Site	Area(s) Covered	# Areas Covered
6	45, 62	2
6	12, 13, 31, 36, 57	5
15	9, 68	2
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
59	6, 24, 61, 69	4
69	23, 56	2
73	4, 40, 43	3
89	64	1
97	2, 3, 44	3
101	17	1
104	39, 58	2
105	34, 63	2
111	20, 21	2
115	1, 32	2
118	38	1
121	26, 46, 48	3
135	16, 55, 59	3
138	18	1
152	25, 41	2
156	27, 65	2
195	14, 15, 51, 52, 53, 60	6
201	22, 54	2

The minimum number of Air Force alert sites needed to cover all 66 Type I areas of interest given 5-minute aircraft launches and a 9 NM per minute aircraft speed is 27. This value is one greater than the minimum number of alert sites required in the comparable run in Model Set I. The p-center objective function value for this model is 141.753 NM which corresponds to the same objective function value in all previous

Model Set I and Model Set IV model runs. The p-median solution for the input parameters of this model does not follow the pattern of the previous results.

As seen in Table 4.20, the p-median solution for the input parameters is 3,740.625 NM. This value is 213.64 NM less than the Model Set I solution for the input values. In all previous Air Force candidate site or Model Set IV runs, the Model Set I p-median solution is less than the Air Force only site solution despite having fewer minimum locations. With the input parameters of a 5-minute launch and 9 NM aircraft speed, the p-median value for the Model Set IV run is less than the Model Set I run with a required minimum of one additional alert site. Although the p-median solution for this particular model run does not follow the trend, the number of binding alert sites does.

The number of binding alert sites in the model is decreased to 4. This value is 3 less than the previous Air Force only site model and is 2 more than the similar Model Set I run. The binding locations as well as the areas causing the binding condition for this run of Model Set IV are shown in Table 4.21.

Table 4.21. Binding Sites for 5-Min. Launch/9 NM per min. Aircraft Speed Model

Binding Sites (4)	Area(s) Causing Binding Condition
47	7, 8, 49
69	23, 56
118	38
49	66

As seen in Table 4.21, not only does the number of binding Air Force only alert sites decrease from previous Model Set IV runs in this model, but the number of areas causing the binding condition decreases as well. This is attributed to the greater critical distance, which permits sites to cover more areas. After Model Set IV results are gathered by

varying launch times at 9 NM per minute aircraft speed, the launch times are adjusted while holding aircraft speed constant at 8 NM per minute

Notional (8-minute) Launch and 8 NM per minute Aircraft Speed Model

The computed critical distance for the launch and aircraft speed parameters in this Air Force only candidate site model is 96 NM. This distance matches the computed critical distance in the comparable Model Set I run. In this derivative of Model Set IV, area 13's critical distance is reduced to 32 NM. This model requires the relaxation of 5 different areas' critical distances as opposed to the 4 relaxed in its Model Set I counterpart. In order for the model to successfully run with the input parameters, the critical distances for areas 33, 37, 38, 44, and 66 must be relaxed. The nearest alert sites to the identified areas are located at respective distances of 102.4721 NM, 98.2689 NM, 141.753 NM, 110.4306 NM, and 125.86 NM. The increased critical distance values of these areas as well as the solutions to this model set are shown in Table 4.22. The Solver™ results corresponding to the independent variable values in this model are included in Appendix E.

Table 4.22. Results for Notional Launch/8 NM per minute Aircraft Speed Model

Table 4.22. Results for No.	otional Launch/8 NM per mii	nute Aircraft Speed Model
<u>Results</u>	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 33 alert sites	66 Type I areas covered	96 NM (areas 1-12,
p-center = 141.753 NM	w/33 alert sites	14-27, 31-32, 34-36, 39-65,
p-median = $3,245.590 \text{ NM}$		and 67-69); 32 NM (area
Avg. dist./p-median =		13); 103 NM (area 33); 99
49.176 NM		NM (area 37); 142 NM
		(area 38); 111 NM (area
		44); and 126 NM (area 66)
<u>Alert Site</u>	Area(s) Covered	# Areas Covered
2	45, 62	2
6	13, 31	2
15	68	1
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
37	61	1
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
57	6	1
69	23, 56	2
73	4, 40, 43	3
83	24	1
89	64	1
92	63	1
96	22, 54	2
97	2, 3, 44	3
101	17	1
104	39, 58	2
105	34	1
111	20, 21	2
115	1, 32	2
118	38	1
121	26, 46, 48	3
131	16, 55, 59	3
138	18	1
148	12, 36, 57	3
151	9	1
152	25, 41	2
156	27, 65	2
177	69	1
195	14, 15, 51, 52, 53, 60	6
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When the critical distance is reduced to 96 NM in Model Set IV, the proven globally minimum number of alert sites to cover all 66 Type I areas of interest is 33. This value is one more than the Model Set I or joint number at this critical distance. The minimum aggregate network distance is 3,245.590 NM, which is 148.236 NM more than the Model I minimum value with the independent variable numbers used in this model. The objective function value of the p-center solution remains unchanged at 141.753 NM. As is the case in many of the previous models, decreasing the critical distance increases the number of binding alert sites.

The Air Force only candidate site model with notional launch times and an 8 NM per minute aircraft speed produces 18 binding sites in its solution set. The binding alert sites and the areas of interest causing the binding condition are presented in Table 4.23.

Table 4.23. Binding Sites for Notional Launch/8 NM per min. Aircraft Speed Model

Binding Sites (18)	Area(s) Causing Binding Condition
57	6
47	7, 8, 49, 50
151	9
6	13
101	17
111	20, 21
69	23, 56
23	33
105	34
148	36
49	37, 66
118	38
73	40
97	44
2	45, 62
37	61
92	63
89	64

This model has 8 more binding alert sites than its Model Set I counterpart. The greater number of binding sites limits network flexibility by giving planners less options in developing an alert network. Each area of interest must be served or covered by the Air Force alert site indicated in Table 4.23 to meet critical distance or response requirements. Once the model is run to determine the optimal alert configuration at 8 NM per minute aircraft speed with notional launch times, the model is run with a one minute reduction in launch time.

7-Minute Launch and 8 NM per minute Aircraft Speed Model

With a 7-minute launch and 8 NM per minute aircraft speed, the critical distance is 104 NM. Area 13's computed critical distance is 40 NM. Both values correspond to the values computed for the similar model in Model Set I. For the model to run, the distance constraints on areas 38, 44, and 66 must be increased to respective critical distances of 142 NM, 111 NM, and 126 NM. The Solver™ results for this run of Model Set IV are located in Appendix E. The LSCP, p-median, and p-center solutions for these input parameters are compiled in Table 4.24.

Table 4.24. Results for 7-Min. Launch/8 NM per minute Aircraft Speed Model

LSCP = 32 alert sites Formula Commonstration Comm	Results	Coverage Scheme	Critical Distance (S_{ij})
p-center = 141.753 NM p-median = 3,324.851 NM Avg. dist./p-median = 50.377 NM w/32 alert sites 14-27, 31-37, 39-43, 45-65, and 67-69); 40 NM (area 38); 111 NM (area 44); and 126 NM (area 66) Alert Site Area(s) Covered # Areas Covered 2 45, 62 2 6 13, 31 2 15 68 1 20 5, 35, 42, 67 4 23 33 1 24 10, 11 2 37 61 1 47 7, 8, 47, 49, 50 5 49 37, 66 2 54 19 1 62 6, 69 2 69 23, 56 2 73 4, 40, 43 3 83 24 1 89 64 1 92 63 1 96 22, 54 2 97 2, 3, 44 3 101 17 1 104 39, 58 2 105 <td< td=""><td></td><td>·</td><td></td></td<>		·	
p-median = 3,324.851 NM Avg. dist/p-median = 50.377 NM and 67-69); 40 NM (area 13); 142 NM (area 38); 111 NM (area 44); and 126 NM (area 66) Alert Site Area(s) Covered # Areas Covered 2 45, 62 2 6 13, 31 2 15 68 1 20 5, 35, 42, 67 4 23 33 1 24 10, 11 2 37 61 1 47 7, 8, 47, 49, 50 5 49 37, 66 2 54 19 1 62 6, 69 2 69 23, 56 2 73 4, 40, 43 3 83 24 1 89 64 1 92 63 1 96 22, 54 2 97 2, 3, 44 3 101 17 1 104 39, 58 2 105 34 1 111			`
Avg. dist./p-median = 13); 142 NM (area 38); 111 NM (area 44); and 126 NM (area 66) Alert Site Area(s) Covered # Areas Covered 2 45, 62 2 6 13, 31 2 15 68 1 20 5, 35, 42, 67 4 23 33 1 24 10, 11 2 37 61 1 47 7, 8, 47, 49, 50 5 49 37, 66 2 54 19 1 62 6, 69 2 69 23, 56 2 73 4, 40, 43 3 83 24 1 89 64 1 92 63 1 96 22, 54 2 97 2, 3, 44 3 101 17 1 104 39, 58 2 105 34 1 115 1, 32 2 116 10, 55, 59 3 138 18 <td< td=""><td>1 *</td><td></td><td></td></td<>	1 *		
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115 1, 32 2 118 38 1 121 26, 46, 48 3 131 16, 55, 59 3 138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	105	34	1
118 38 1 121 26, 46, 48 3 131 16, 55, 59 3 138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	111	20, 21	2
121 26, 46, 48 3 131 16, 55, 59 3 138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	115	1, 32	2
131 16, 55, 59 3 138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	118	38	1
131 16, 55, 59 3 138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	121	26, 46, 48	3
138 18 1 148 12, 36, 57 3 151 9 1 152 25, 41 2 156 27, 65 2	131		3
151 9 152 25, 41 156 27, 65			
151 9 152 25, 41 156 27, 65		12, 36, 57	3
152 25, 41 2 156 27, 65 2			
156 27, 65 2		25, 41	2
			l .

The Air Force only site LSCP solution for the input parameters is a global minimum of 32 alert sites to cover all of the required areas of interest. As in previous Air Force only alert site models, the optimal solution requires one more alert site than the Model Set I run with the same critical distance and one less site than the notional launch/8 NM per minute aircraft speed model in Model Set IV. The p-center objective function value is 141.753 NM, which is identical to the comparable Model Set I run as well as the previous run in Model Set IV. Although the LSCP and p-center solutions are somewhat predictable, this is not the case for the p-median solution in this particular model.

The minimum aggregate network distance given Air Force only alert sites with a 7-minute launch and 8 NM per minute aircraft speed is 3,324.851 NM. The aggregate network distance exceeds its Model Set I counterpart by 173.736 NM. The p-median solution for this model is different because it does not follow the developing pattern for models within the same model sets possessing the same minimum number of alert sites or LSCP solutions. Specifically, in all previous runs within the same model sets that have the same LSCP solution value, the p-median solutions have also been identical. In all Model Set I runs with an LSCP value of 31, the p-median objective function is 3,151.115 NM. Similarly, in previous Model Set IV runs with an optimal LSCP value of 32, the pmedian objective function is 3,315.869 NM. This p-median objective function value does not occur in this model. The different p-median solution is explained by a 4 NM difference in critical distance, which causes a difference in the coverage of areas 39 and 58 between the two models. Subsequently, this model has a different set of 32 sites to cover the 66 Type I areas. The difference in coverage makes the optimal p-median solution in this model 8.982 NM greater than the notional launch/9 NM per minute

aircraft speed model in Model Set IV. Although the p-median solutions for the two Model Set IV models differ, the number of binding alert sites do not.

Given the input parameters of 7-minute launch and 8 NM per minute aircraft speed, the model has 17 binding alert sites. This is identical to the notional launch/9 NM per minute aircraft speed model in Model Set IV, but is 8 sites greater than the Model Set I run with the same input parameters. The areas causing the binding conditions for this model are also identical to the previously mentioned run in Model Set IV. The reader is referred to Table 4.17 for a listing of the binding alert sites and the allocation of the areas to the sites for this model run.

6-Minute Launch and 8 NM per minute Aircraft Speed Model

Applying a 6-minute launch and 8 NM per minute aircraft flight speed in the Air Force only model produces a critical distance of 112 NM. The distance corresponds to the value computed for the application of these parameters in Model Set I. The critical distance for area 13 is 48 NM. Distance constraints for areas 38 and 66 must be relaxed to 142 NM and 126 NM to allow the model to run. Snapshots of the actual Solver™ results for this run of Model Set IV are located in Appendix E. The LSCP, p-center and p-median solutions to this model as well as alert site area coverage are identical to the solutions obtained for the notional launch and 9 NM per minute aircraft speed model in this model set. The reader is referred to Table 4.16 for a breakdown of the results. The computed critical distances for this model corresponds to the values discussed earlier in this paragraph and do not replicate the values in Table 4.16.

As seen in Table 4.16, the minimum number of Air Force only sites needed to cover the 66 Type I areas of interest is 32. The Air Force only solution requires an

additional site than is required in the joint or Model Set I LSCP solution with the same input parameters. The p-median solution of 3,315.869 NM is 164.754 NM greater than the Model Set I p-median objective function value, which covers the demand with 31 sites as opposed to 32. The p-center objective function value for this run and the comparable run in Model Set I are identical. The p-center objective function value for both models is 141.753 NM. Although the LSCP, p-median, and p-center solutions for this model are identical to the notional launch/9 NM per minute model presented earlier in this model set, the number of binding alert sites differ between the two models.

This model requires 15 binding alert sites while the notional launch/9 NM per minute model of Model Set IV requires 17. The allocation of the areas and the binding alert sites for this Air Force only model are identical to the results that are presented in Table 4.17 with the exception that alert sites 6 and 89 are not binding in this model. The 15 binding Air Force only alert sites in this model exceed the Model Set I results for the same input parameters by 7 binding sites. As previously discussed, a greater number of candidate sites presents a wider range of coverage options and therefore less binding alert sites. After Model Set IV is run with a 6-minute launch and 8 NM per minute aircraft speed, the model set is optimized with a launch time of 5 minutes while holding the 8 NM aircraft speed constant.

5-Minute Launch and 8 NM per minute Aircraft Speed Model

As in Model Set I, these parameters produce a critical distance of 120 NM. Also, the critical distance for area 13 is 56 NM. The distance constraints for areas 38 and 66 must be relaxed to 142 NM and 126 NM to allow the model to run. Snapshots of the Solver™ results for this model are included in Appendix E. Results of the model runs,

including alert sites, areas covered, LSCP, p-median, and p-center, are shown in Table 4.25.

Table 4.25. Results for 5-Min. Launch/8 NM per minute Aircraft Speed Model

Table 4.25. Results for 5-Min. Launch/8 NM per minute Aircraft Speed Model		
Results	Coverage Scheme	Critical Distance (S_{ij})
LSCP = 30 alert sites	66 Type I areas covered	120 NM (areas 1-12,
p-center = 141.753 NM	w/30 alert sites	14-27, 31-37, 39-65, and
p-median = $3,410.175 \text{ NM}$		67-69); 56 NM (area 13);
Avg. dist./p-median =		142 NM (area 38); and 126
51.669 NM		NM (area 66)
<u>Alert Site</u>	Area(s) Covered	# Areas Covered
2	45, 62	2
6	13, 31	2
15	9, 68	2
20	5, 35, 42, 67	4
23	33	1
24	10, 11	2
37	61	1
47	7, 8, 47, 49, 50	5
49	37, 66	2
54	19	1
62	6, 69	2
69	23, 56	2
73	4, 40, 43	3
83	24	1
89	64	1
96	22, 54	2
97	2, 3, 44	3
101	17	1
104	39, 58	2
105	34, 63	2
111	20, 21	2
115	1, 32	2
118	38	1
121	26, 46, 48	3
131	16, 55, 59	3
138	18	1
148	12, 36, 57	3
152	25, 41	2
156	27, 65	2
195	14, 15, 51, 52, 53, 60	6
	, , , , , - ,	1

This model produces a LSCP solution of a minimum of 30 Air Force affiliated alert sites to cover all 66 Type I areas of interest. This exceeds the Model Set I solution with the same input parameters by one site. The p-center objective function value of 141.753 NM for the model is identical to its Model Set I counterpart and all of the other Model Set I and Model Set IV runs. The p-median aggregate network distance is 3,410.175 NM, which is 91.279 NM greater than the aggregate network distance solution produced in Model Set I. Model Set I achieves its total coverage distance with one less site.

Although the LSCP, p-median, and p-center solutions for this run of Model Set IV are relatively close to the results generated in Model Set I for the same input values, the number of binding locations between the two model runs are markedly different.

For a 5-minute launch and 8 NM per minute aircraft speed, Model Set IV shows 10 binding Air Force only alert sites. Conversely, Model Set I reveals 4 binding joint alert sites with identical parameters. The binding sites as well as the areas of interest causing the binding conditions are presented in Table 4.26.

Table 4.26. Binding Sites for 5 Min. Launch/8 NM per min. Aircraft Speed Model

Binding Sites (10)	Area(s) Causing Binding Condition
47	7, 8, 49, 50
101	17
111	20
69	23, 56
148	36
49	37, 66
118	38
97	44
2	45, 62
37	61

The increase in critical distance from the previous Model Set IV run with a 6-minute launch results in a 5 binding alert site reduction. Additionally, given the increase in

critical distance, the number of areas causing binding conditions is reduced as has been shown in many of the previous models.

Sensitivity Analysis

After running the different model sets with varying independent variable values, sensitivity analysis is conducted on the different model configurations to see how sensitive the previously generated network solutions are to changes in critical distance. Also, sensitivity analysis is done to compare the overall performance of the joint site solutions in the Model Set I runs with the Air Force only site results generated in Model Set IV. First, the LSCP solution values for Model Sets I and IV are mapped according to critical distance. Second, the p-median aggregate network distances for Model Sets I and IV are presented corresponding to critical distance. Third, average site response time is computed for each critical distance for all Model Set I and Model Set IV runs. Fourth, LSCP solutions for Model Sets I and IV are presented by critical distance to show percentages of areas covered as sites are incrementally removed from the optimal network. Finally, the section ends with the presentation of common and binding alert sites that are present in all Model Set I, Model Set IV, and both runs when aggregated.

LSCP Solutions Mapped Against Critical Distance-Model Set I vs. Model Set IV

Figure 4.1 maps the minimum number of required alert sites generated in the runs
for Model Sets I and IV against critical distance.

Comparison of Needed AF Only and Joint Sites by Varying Launch Times and Aircraft Speeds

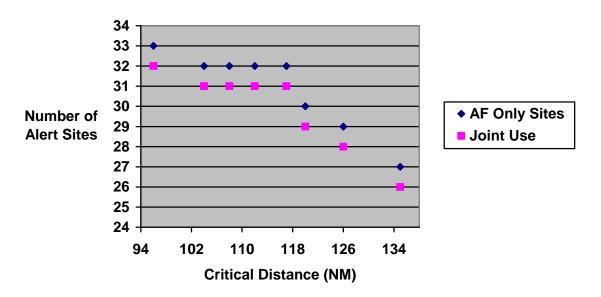


Figure 4.1. Minimum Number of Alert Sites Required per Critical Distance

As seen in Figure 4.1, the joint solution or Model Set I solution is always one less site than is generated from the Air Force only or Model Set IV configuration. Also, the areas of interest that require coverage are such that 31 and 32 sites are optimal minimums for computed critical distances between 104 NM and 117 NM. This is significant because this minimum number of sites covers 50% of the scenarios or critical distances generated in the model sets. This figure also demonstrates that the relationship between the minimum number of alert sites and the computed critical distance is an inverse one with the exception of the straight line noted. After mapping the LSCP solutions against critical distances, the p-median solutions for Model Sets I and IV are compared against critical distance.

P-Median Solutions versus Critical Distance--Model Set I and Model Set IV

The p-median solutions for Model Sets I and IV do not follow the same relationship as shown for the LSCP solutions in Figure 4.1. While the LSCP solutions show an inverse relationship with critical distance, the p-median solutions have a direct one. The direct relationship is demonstrated in Figure 4.2.

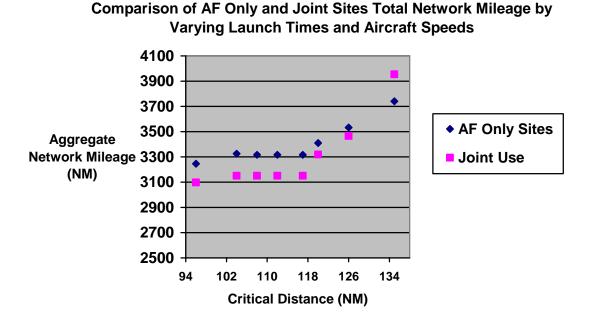


Figure 4.2. P-Median Solutions for Different Critical Distances

As seen in Figure 4.2, the minimum aggregate network distance increases with an increase in critical distance. Also, the straight line between 104 NM and 117 NM shows that the model sets produce nearly the same p-median solution across the range. This is significant because the minimum network distance remains at the indicated minimum throughout a wide range of parameters. Figure 4.2 also shows that at a critical distance greater than 126 NM the joint use or Model Set I p-median solution starts to increase past the Air Force only alert site or Model Set IV solution. Even though the joint use

configuration covers its areas with one less site across the spectrum than do the Model Set IV runs, the Air Force only p-median aggregated distance at critical distances exceeding 126 NM is less than the joint use solution's aggregate distance. This identifies an area where defense planners need to balance the trade-offs between overall network distance and number of alert sites.

Alert Site Average Response Time Comparison

Alert site average response time is computed by using equation 22 from Chapter 3. The minimized aggregate network distance or p-median solution is used to compute the average distance traveled per selected site. This calculation enables the computation of average response time per site. The site average response times for different computed critical distances are shown in Figures 4.3 and 4.4. Figure 4.3 shows the average response time for the joint use or Model Set I solutions and Figure 4.4 includes the average response times for the Air Force affiliated or Model Set IV solutions.

Joint Use Model Sets Alert Site Average Response Time

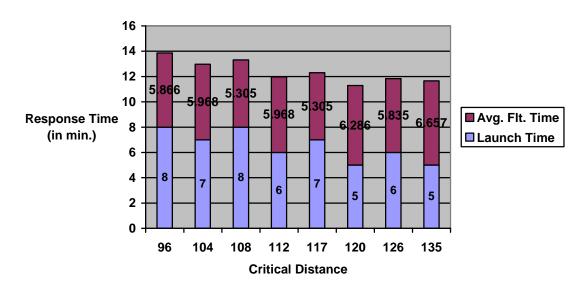


Figure 4.3. Joint Use Alert Site Average Response Time by Critical Distance

Air Force Only Model Sets Alert Site Average Response Time

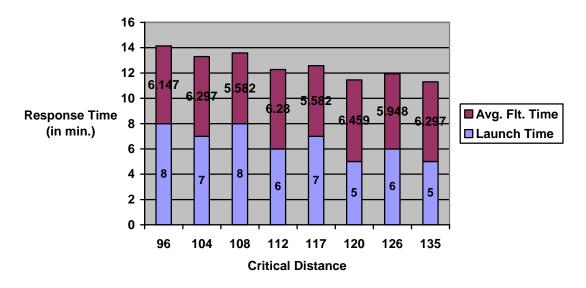


Figure 4.4. Air Force Only Alert Site Average Response Time by Critical Distance

As in previous results, the joint use model produces better results with one less alert site until critical distances exceed 126 NM. At a critical distance of 135 NM the Air Force only affiliated site model's average response time per site solution is .36 minutes better per site than the joint use solution. Also, Figures 4.3 and 4.4 demonstrate that average alert site response time is highly dependent on aircraft launch time. In most cases, except those with 5-minute aircraft launch times, aircraft launch time exceeds average flight time from an alert site to an area of interest. Not shown in Figures 4.3 and 4.4 is the affect that aircraft speed has on response time. Aircraft speed directly affects average flight time because the p-median average distance per site for the computed critical distance is divided by aircraft speed to obtain average flight time in minutes.

Although it appears in Figures 4.3 and 4.4 that a larger critical distance produces a faster response time, this is not exactly the case. Defense leaders must take into account

the feasibility of achieving 5- and 6-minute launch times at every site in the network before committing to a coverage solution with a computed critical distance of 120 NM, 126 NM, and 135 NM. Also, the feasibility of achieving a 9 NM per minute aircraft speed must be examined. The computed average site response times in the range of computed critical distances between 104 NM and 117 NM for the joint use or Model Set I solutions and the Air Force only or Model Set IV solutions represent more conservative estimates. Each configuration represents managerial trade-offs that require evaluation. A total average response time per site by critical distance comparison is shown in Table 4.27. Aircraft launch times and aircraft flight speeds used to compute the respective critical distances are included in the table.

Table 4.27. Comparison of Average Alert Site Response Times for Model Sets I and IV by Critical Distance

Critical	Aircraft	Aircraft Flight	Joint Use	AF Only	Delta (AF
Distance (S_{ij}	Launch Time	Speed (AS in	Site Avg.	Site Avg.	Only –
in NM)	(ACLT in	NM per	Resp. Time	Resp. Time	Joint Use
	min.)	minute)	(min.)	(min.)	(in min.)
96	8	8	13.866	14.147	.281
104	7	8	12.968	13.297	.329
108	8	9	13.305	13.582	.277
112	6	8	11.968	12.28	.312
117	7	9	12.305	12.582	.277
120	5	8	11.286	11.459	.173
126	6	9	11.835	11.948	.113
135	5	9	11.657	11.297	36

Table 4.27 demonstrates that the joint use alert site average response time per site is .113 minutes to .329 minutes better than the Air Force only affiliated or Model Set IV solutions. The joint use alert site average response times are better at the indicated critical distances with one less required alert site. In the range identified in previous sections as being insensitive to changes in input parameters, the Model Set I or joint use

site average response time is .277 minutes to .329 minutes better per site than the Air Force only or Model Set IV site average response time solution. Also, as in previous models, the Air Force only solution is better than the joint use solution at a computed critical distance of 135 NM; however, the Air Force only solution is achieved with one more alert site than the joint use value.

LSCP Solution Site Sensitivity with Optimal Coverage Network

LSCP solutions for Model Sets I and IV are presented by critical distance to show percentages of areas covered as one site is removed incrementally from the optimal network. The optimal network configuration for each critical distance is presented earlier in this chapter. Figures 4.5-4.9 present the percentage of areas of interest covered with a specific number of alert sites given different critical distances for the optimal joint use and Air Force only solution sets.

Coverage Sensitivity with Critical Distance = 96 NM

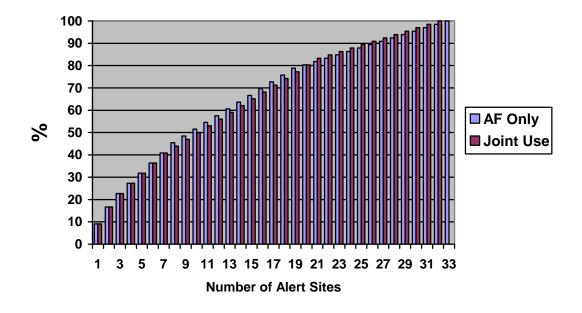


Figure 4.5. Percentage of Areas Covered at 96 NM Critical Distance

As seen in Figure 4.5, at a critical distance of 96 NM the joint use or Model Set I solution can cover approximately 2 percent more of the areas of interest from 32 down to 21 sites. At 19 sites, the Air Force only solution can cover approximately 3 percent more of demand than the joint solution down to 7 sites when both solutions can cover equal percentages of the areas of interest. After examining the sensitivity at a critical distance of 96 NM, the percentage of areas covered is looked at for critical distances of 104 NM, 108 NM, 112 NM, and 117 NM.

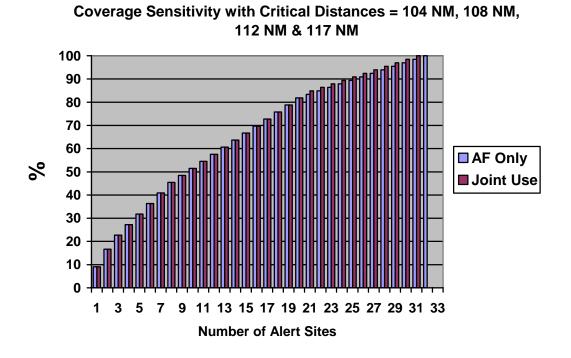


Figure 4.6. Percentage of Areas Covered at 104 NM, 108 NM, 112 NM, & 117 NM Critical Distances

Figure 4.6 shows the percentage of areas covered for the indicated critical distances. From 31 down to 21 alert sites the joint use or Model Set I solution is capable of covering a greater percentage of the areas of interest by approximately 2 percent. At

20 alert sites down to 1 site the models perform equitably or cover an equal portion of the areas of interest. These percentages are significant because they identify potential trade-off costs for managers. For example, if resources are limited to 21 sites, then, the joint use solution would be capable of covering more demand at these critical distances than the Air Force only solution. Once the solution sensitivity is evaluated for critical distances from 104 NM to 117 NM, the sensitivity is examined for a critical distance of 120 NM. These results are presented in Figure 4.7.

Coverage Sensitivity with Critical Distance = 120 NM

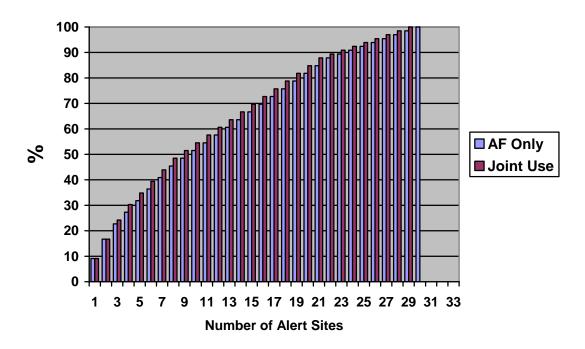


Figure 4.7. Percentage of Areas Covered at a Critical Distance of 120 NM

At a critical distance of 120 NM, the joint use or Model Set I alert sites cover 1.5 percent more of the areas of interest than the Air Force only solution from 29 down to 22 sites. At an alert site level of 21 sites, the joint use or Model Set I solution covers 3 percent more of the areas of interest than the Model Set IV or Air Force only solution.

This disparity continues until a level of 3 sites, where the joint use solution covers a higher percentage of demand than the Air Force only solution by 1.5 percent. At levels of 1 and 2 sites, given the 120 NM critical distance, Model Set I and Model Set IV cover equal percentages of demand. Once the percentage of areas covered is examined for a critical distance of 120 NM, the sensitivity is examined for a critical distance of 126 NM.

Coverage Sensitivity with Critical Distance = 126 NM

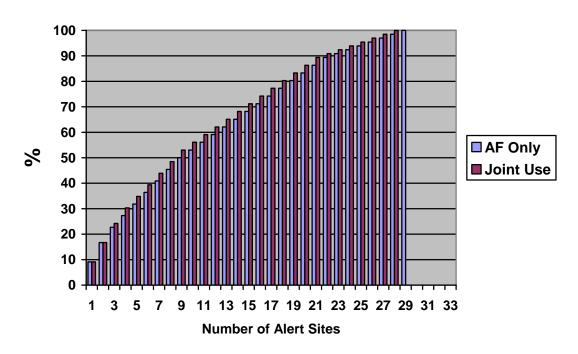


Figure 4.8. Percentage of Areas Covered at a Critical Distance of 126 NM

Figure 4.8 presents the sensitivity analysis for the Model Set I and Model Set IV solutions given a critical distance of 120 NM. At a level of 28 alert sites, the joint use or Model Set I network is capable of covering 1.5 percent more of the areas of interest than the Model Set IV or Air Force only network. This disparity continues until a level of 21 sites, where Model I covers 3 percent more of the demand than Model Set IV. The 3 percent gap exists until a level of 3 alert sites where the gap drops again to 1.5 percent

and is subsequently closed for levels of 2 and 1 sites. After exploring the coverage sensitivity for a critical distance of 126 NM, the percentage of areas covered is examined for a critical distance of 135 NM. These results are presented in Figure 4.9.

Coverage Sensitivity with Critical Distance = 135 NM

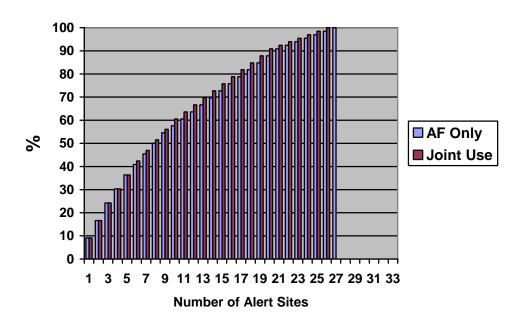


Figure 4.9. Percentage of Areas Covered at a Critical Distance of 135 NM

At a critical distance of 135 NM the joint use solution covers 1.5 percent more of the demand than the Air Force only network from an alert site level of 28 to 21. At a level of 20 down to 10 alert sites, the joint use solution covers 3 percent more of the areas of interest than the Air Force only network. From a level of 9 to 6 alert sites, the joint use solution is capable of covering 1.5 percent more of the areas of interest than the Air Force only solution. When the level of alert sites reaches 5, either network is capable of covering the same percentage of demand. After evaluating network sensitivity given critical distances, common and binding alert sites are identified in Model Sets I and IV to show sites that are insensitive to all model treatments.

Common and Binding Joint Alert Sites to All Model Set I Configurations

The common joint use and binding alert sites that are found in every Model Set I configuration are presented in Table 4.28. These sites are insensitive to the varying of aircraft launch times between 5-8 minutes and changing the aircraft speed between 8-9 NM per minute. The binding sites must be part of every presented model solution in Model Set I. While part of every solution set, the common sites do not necessarily have to be in the solution.

Table 4.28. Common and Binding Joint Alert Sites for All Model Set I Solutions

Common Joint Alert Sites (19)	Binding Alert Sites (2)
1, 2, 20, 23, 24, 47, 49, 54, 69, 73, 105, 111, 118,	47, 118
121, 138, 152, 156, 159, 170	

Table 4.28 shows that 19 sites are common to each Model Set I solution. The table also shows 2 joint alert sites that are binding in every different model. Knowing these common and binding locations gives defense leaders an idea of where the more strategic alert sites are located in the CONUS. ACC and First Air Force can use the information to work with other services to build the best overall alert network. Finally, defense planners who identify bases for realignment and closure can use this information to make better decisions, because once a strategic location is closed or turned over it is very difficult to recoup. After examining the sites for the joint use model, the common and binding sites are considered in the Air Force only or Model Set IV solutions.

Common and Binding Air Force Sites to All Model Set IV Configurations

All Air Force only or Model Set IV models contain a set of alert sites that are common to all different critical distances. These alert sites are insensitive to changes in the input values of launch time and aircraft speed used in this research. In addition to the

common alert sites in all solutions of Model Set IV, specific binding sites must be selected in all solution sets due to coverage requirements. The common and binding sites to the Air Force only candidate site model set are shown in Table 4.29

Table 4.29. Common and Binding Air Force Alert Sites for Model Set IV Solutions

Common Air Force Sites (23)	Binding Alert Sites (4)
2, 6, 15, 20, 23, 24, 47, 49, 54, 69, 73, 89, 97, 101, 105,	47, 69, 118, 49
111, 115, 118, 121, 138, 152, 156, 195	

All Model Set IV runs contain 23 common alert sites as well as 4 binding alert sites shown Table 4.29. As discussed in the previous paragraph, knowing the alert sites that are advantageously located regardless of input parameters can help defense planners in evaluating the overall air defense network. If an Air Force affiliated alert site network is sought, these alert sites would be good candidates based on the identified areas of interest. Also, the alert site locations can be factored into Air Force infrastructure decisions. After identifying the common and binding sites for Model Sets I and IV individually, the common and binding sites are identified for the two combined model sets.

Common and Binding Alert Sites to Model Set I and IV with Type II Sites

This section presents the common and binding sites to all Model Set I and IV solutions as well as shows the Type II binding sites shown in Model Sets II and III. The sites are presented in Table 4.30.

Table 4.30. Common and Binding Sites for Model Sets I and IV with Type II Sites

Common Alert Sites (16)	Binding Alert Sites (2)	Binding Type II Sites (4)
2, 20, 23, 24, 47, 49, 54, 69, 73,	47, 118	135, 150, 98, 67
105, 111, 118, 121, 138, 152, 156		

There are 16 alert sites common to all Model Set I and IV solutions. Two of the sites are binding in all of those mentioned solutions. Finally, as discussed in the Model Set II and

Model Set III sections, all alert sites covering the Type II areas of interest are binding.

As previously mentioned, knowing the common alert sites to all models given the different critical distances has numerous managerial uses in the Air Force as well as the DoD. After gathering the results for all of the models and conducting sensitivity analysis on the results, the research questions are revisited.

Research and Investigative Questions

The primary research question for this thesis was "What are the optimal strip alert locations in the Continental United States for aircraft in support of homeland defense of the United States?" In order to answer the primary research question, four investigative questions were answered over the course of this research.

Investigative question one "What is the history of the alert network (Cold War to present)?" was addressed in Chapter 2. The air defense network of the United States has undergone fundamental changes since the Cold War. These changes have included different types of aircraft, a different structure of the alert network, a change in adversary, and a change in philosophy. The United States can no longer look outward with its alert network for the Soviet Union and manned bombers. The United States must take an inward as well as outward approach to air defense in order to combat unconventional threats such as terrorism and protect its borders from intruders.

The second investigative question, "What are the alert system objectives and their relative importance in the overall air defense network?" was answered in Chapter 3. The equally important objectives are as follows: 1. minimize aircraft response time; 2. cover all areas of interest with at least one site; 3. minimize the number of strip alert locations;

4. minimize overall or average distance per network location; and 5. minimize maximum travel time for an aircraft at any location in the network. These objectives were obtained from the ACC Department of Homeland Security and the First Air Force Air Operations Center.

Investigative question three "What is the best method for solving the strip alert network problem and what are the critical model parameters leading to a specific modeling method?" is discussed in Chapters 2-3. The methods are presented in Chapter 2, but the decision to use the location set covering problem (LSCP), p-median algorithm, and p-center problem occurs in Chapter 3. The selection of these methods corresponds to the network objectives. The p-median solution addresses objectives 1 and 4. The LSCP meets the needs of objectives 2 and 3. Finally, the p-center problem is proven adequate at solving objective 5. The parameters leading to technique selection include aircraft type, launch and operating characteristics, as well as candidate alert site requirements, the list of the areas of interest, and response requirements to the areas of interest. These parameters, coupled with overall system objectives, led to the selection of optimization and the aforementioned location modeling techniques to solve the problem.

The fourth investigative question, "How do optimal network solutions change when adjustments are made to critical model parameters?" is answered in Chapter 4. Changes in aircraft launch and operating characteristics have minimal affect on the optimal network solutions for computed critical distances of 104 NM, 108 NM, 112 NM, and 117 NM. Optimal network solutions for minimum number of sites, minimum response time, minimized aggregate network distance, and minimized maximum distance

remain relatively constant within the aforementioned range. Outside of this range of critical distances, the affects are more profound.

"What are the optimal strip alert locations in the Continental United States for aircraft in support of homeland defense of the United States?" can be answered. This chapter shows that the optimal strip alert locations depend on available alert sites and critical distance. If the DoD decides to use a network with joint sites (Army, Navy, Air Force, and Marines), the solutions for 31 sites in Model Set I cover computed critical distances from 104 NM - 117 NM with a minimum network distance or p-median aggregated distance of 3,151.115 NM and a p-center objective function value of 141.753 NM. Also, the average site response times range from 11.968 minutes to 13.305 minutes for this model set. The actual alert sites and the way the areas are allocated to the sites are found in Table 4.1 and the response times can be located in Figure 4.3.

Conversely, if defense leaders opt to make the network strictly an Air Force alert site operation (Air Force Reserve, ANG, and active duty), the different solutions for 32 alert sites in Model Set IV also cover computed critical distances between 104 NM - 117 NM with p-median solutions of 3,315.869 NM for critical distances greater than 106 NM and a value of 3,324.851 NM for critical distances 105 NM and less. The optimal alert site and coverage configuration for distances greater than 106 NM is found in Table 4.16 and the site and coverage configuration for critical distances 106 NM and less is found in Table 4.24. The Air Force only model set produces average alert site response times between 12.28 minutes to 13.582 minutes within the previously mentioned range of computed critical distances. The average site response times are shown in Figure 4.4.

Both the joint use and Air Force only solutions are insensitive to parameter changes within the range of computed critical distance between 104 NM - 117 NM. A tertiary objective was to find the optimal locations to deploy aircraft to cover non-permanent Type II areas of interest.

Model Sets II and III revealed that the optimal alert sites for the Type II areas of interest are at locations 135, 150, 98, and 67. These alert sites cover the respective Type II areas 28, 29, 30, and 70. Each of these areas of interest is capable of being served by only one alert site each given notional launch times and 9 NM per minute aircraft speed. Therefore, each of these sites is binding given the input parameters. Ultimately, the optimal network depends on which approach the DoD decides to use (joint or Air Force only candidate alert sites). However, this research presents solutions to either scenario and offers optimal alert sites to cover the Type II areas interest on an as-needed basis.

Summary

This chapter presents the results of the different model set runs developed in Chapter 3. Model Sets I and IV are run by varying the critical distance through the manipulation of the independent variables of aircraft launch time and aircraft speed. Solutions to the LSCP, p-median, and p-center problems are presented for all model runs. Additionally, the Type I area of interest allocation to each alert site in the solution set for each model run is presented. Model Sets II and III are run once with the notional launch times and maximum aircraft speed to determine the optimal locations to cover Type II areas of interest. The optimal sites to cover the Type II areas of interest are presented as well possible alert sites that could cover Type I areas as well. After presenting the results

of all model runs, sensitivity analysis is done to demonstrate how responsive the results of Model Sets I and IV are to changes in the independent variable values and identify managerial trade-offs. Next, common and binding sites that are insensitive to changes to input parameters in this research are presented. Finally, the chapter ends with the answering of the investigative and primary research questions. In Chapter 5, the managerial implications, recommendations, and limitations of this research are presented along with recommendations for further study.

V. Conclusions and Recommendations

Introduction

Chapter 1 provided the foundation for this research effort by providing the justification for building a mathematical model to optimize the location of strip alert sites. The events of 11 September 2001 caused a fundamental shift in the way that the United States conducts homeland defense and the air defense mission. Terrorism caused defense leaders to re-evaluate the air defense mission and the air defense alert network. Instead of an exclusive outward looking, border defense strategy, the United States needs to be cognizant of internal areas within the country requiring air defense. Given smaller budgets and the downsizing of the Air Force, the new alert network must be efficient as well as effective.

Chapter 2 presented a review of the relevant literature to the alert network problem. It discussed the history of the strip alert network from Cold War to present and the evolution, suitability, taxonomy, and application of location modeling techniques. Since the end of the Cold War, the threats confronting the United States shifted from the robust, Soviet-manned bomber fleet to the unconventional foe of terrorism. This change in adversary has driven a need for a change in the way in which the air defense mission and strip alert network is organized. Also due to fiscal policy and the fall of the Soviet Union, the numbers of alert aircraft and alert sites have dropped dramatically since the height of the Cold War. With the unpredictably of the modern threat environment, a premium is placed on optimal positioning of alert sites and aircraft.

Many different location modeling techniques were presented to aid in selection of the best method to solve the alert site positioning problem. By using location modeling techniques, the DoD and Air Force can implement an optimal alert network in the most efficient and effective manner possible. Location modeling was shown to be suitable at siting a host of different types of resources, including fire trucks and ambulances.

Finally, different solution methods for solving location problems were presented to aid in solution method selection for the strip alert network problem.

Chapter 3 provided a discussion of the methodology used in this research. It presented the data collection process and objectives of the overall alert network. The mathematical formulation of the location analysis techniques used in this thesis were presented along with critical model parameters. Also, network operation and candidate site assumptions were presented to simplify the problem so that optimization could be used as the solution method. Finally, the four model sets constructed for analysis as well as the areas and methods used in sensitivity analysis were presented.

The results of this research effort were presented in Chapter 4. The results of the different model set runs were reviewed to determine the optimal strip alert network configuration. The optimal alert site network and coverage schemes were presented for each model run. Also, binding sites were presented for each model set. Sensitivity analysis was conducted to establish how sensitive the model results were to changes in input parameters as well as to identify which alert sites remained in the solution set despite changing the input parameters of the models. The chapter concluded with revisiting and answering the five investigative questions and the overarching research question.

This chapter briefly summarizes the results from Chapter 4 and explores inferences that can be drawn from the results. Additionally, the chapter presents the managerial implications of the model results and this research effort. Recommendations for action are given, to include sequence of action. Finally the limitations of the research are presented followed by recommendations for future research.

Findings

The optimal alert network depends on desired candidate sites, areas of interest requiring coverage, and the input parameters for the model. If all 202 joint use sites meeting the runway distance requirements set by the ACC Department of Homeland Security are used, the areas of interest explored in this research can be most efficiently covered by a minimum of 31 alert sites at a minimum aggregate network distance of 3,151.115 NM. The alert sites and specific areas covered are presented in Table 4.1. This solution is optimal for critical distances computed in the range of 104 NM - 117 NM and is the least sensitive to changes in input parameters. Average response time varies within the mentioned range from 11.968 minutes to 13.305 minutes. A comparison of response times is located in Table 4.27. The critical distances are computed by using different launch and aircraft speed parameters. Generally, if the critical distance is increased then the minimum number of sites decreases and the aggregate network distance increases within the range of parameters used in the models. These relationships are presented in Figures 4.1 and 4.2.

The optimal alert network configuration also changes when Army, Navy, and

Marine Corps sites are removed from the candidate site list. Using Air Force affiliated

only alert sites increases the minimum number of alert sites to 32 sites. These 32 sites are capable of covering the areas of interest at an aggregate network distance of 3,324.851 NM within critical distances computed in the range of 104 NM - 105 NM. A minimum of 32 sites is also capable of covering all the areas of interest in this research at computed critical distances within the range of 106 NM - 117 NM at a reduced aggregate network distance of 3,315.869 NM from the previous range. The 32 site Air Force only solution is the least sensitive to changes in input parameters and is optimal given a wide a range of launch and aircraft speed scenarios. The alert sites for the optimal Air Force only candidate site networks are presented in Tables 4.16 and 4.24. As in the joint site model set, if the critical distance is increased then the minimum number of sites decreases and the aggregate network distance increases within the range of parameters used in the models. The average site response time for the Air Force only solution varies between 12.28 minutes to 13.582 for the computed critical distances in the noted ranges. A comparison of joint use and Air Force only average site response times is found in Table 4.27. This applies to all parameter combinations except the case noted above. In this instance, when the critical distance reaches 106 NM the aggregate solution actually decreases. These relationships are presented in Figures 4.1 and 4.2.

Certain alert sites are common to all model solutions; however, are not required to be in the solution sets. These sites are important because no matter how the input parameters change in this research, the advantageous location of the sites causes them to always be in the solution. Also, certain sites must be in the solution and are considered binding. Between the joint and Air Force only candidate alert site solutions, there exist 16 common alert sites, 2 binding Type I sites, and 4 binding Type II area sites. These sites

are shown in Table 4.30. All Type II areas cause a binding alert site and represent highly variable, non-repetitive demand.

When the Type I and Type II areas were aggregated into a single model, it was discovered that removing non-binding sites from the solution set caused the minimum aggregate network distance to increase. Additionally, all Type II areas used in this research caused a binding alert site and represented highly variable, non-repetitive demand.

It was also shown that the network is incapable of meeting the response requirements to area of interest 38 in every configuration. Therefore, the critical distance for area 38 had to be increased in every model set run in order for a feasible solution to be generated. Also, in many of the model runs, the critical distance for area 66 had to be relaxed in order for the model run. These findings are critical because the absence of an alert site within the critical distance for the two sites limits coverage options. Ultimately, the optimal alert network configuration depends on what defense planners constitute as suitable sites, what areas of interest they wish to cover, and what aircraft response is deemed acceptable.

Managerial Implications

Defense leaders at NORAD, the Headquarters United States Air Force, Air Combat Command, and First Air Force can take the modeling methods as well as the results generated in this research to make real world decisions. The use of mathematical models allows the user to find solutions that are not obvious. The models can also be updated when new areas of interest (Type I or Type II) require coverage or new candidate

alert sites are introduced to produce a new optimal network. Finally, numerous what-if scenarios can be posed to the models to see how the optimal network configuration is affected. The location models used in this research could also be used to optimally base deployed aircraft based on desired proximity to the enemy, response requirements, and desired target coverage. In addition, the model produces solutions to economize the use of force to prevent excessive overlap of resources during contingencies. The models can be used in a greater-than distance scenario creating a p-dispersion model. For instance, if the critical distance were used with a greater-than constraint instead of a less-than constraint, policy makers could use the model to base aircraft away from the effective range of an adversary's conventional or nuclear weapons. Subsequently, in this scenario a greater aggregate p-median value would be desired.

Defense leaders can also take the actual results of the models generated in this research and make informed decisions about the current and future strip alert network. The different network configurations can be used based on historical launch times and differing aircraft speeds. Also, leaders who make decisions about Base Realignment and Closure actions can use the results of the models in the evaluation of a base's suitability for supporting the air defense mission. Additionally, decisions can be made based on limited funding for alert sites. Specifically, planners can use the results of the sensitivity analysis to see how much demand would not be covered given limited funding for a fixed number of sites. Finally, defense leaders can use the results from the models to identify areas that might be better served by non-continuous CAPs based on intelligence information rather than constant strip alert posture due to proximity to nearest alert site (areas 38 and 66).

Recommendations

It is the recommendation of this research that the areas identified in the model runs requiring critical distance relaxation be evaluated to see if relaxation of the distance constraints produces an acceptable response time. If the relaxation creates an unacceptable response time, this research recommends covering these areas with continuous or non-continuous CAPs based on threat assessment and removing them from the model runs. Other options include allowing candidate sites not meeting the minimum runway restriction to be considered as well as the possibility of new construction. New construction is undoubtedly the least attractive alternative. After removing the areas covered by CAPs instead of the strip alert network, the model should be re-solved with desired parameters to find the new optimal network. If the relaxation of the distance constraints are acceptable, this research recommends evaluating the alert sites in the desired network configuration for suitability at handling the alert mission. This would require an analysis of infrastructure, tanker availability, and required funding to correct any deficiencies.

It is also recommended that policy makers explore the feasibility of incorporating joint alert sites into the alert network. As seen in the model runs, the optimal network configuration is consistently better in minimum number of locations and optimal network distance when joint sites are used as opposed to Air Force affiliated sites only up to a critical distance of approximately 126 NM. For expected critical distance exceeding 126 NM, leaders should evaluate the trade-offs between the two networks.

The models also demonstrate that a joint site network produces more options in terms of coverage as well fewer binding alert sites and better average response time in the solution sets than the Air Force only model set. Whether a joint network is implemented or not, this research recommends that planners closely examine the network configurations presented in the Findings Section of this chapter, because both solutions are the least sensitive to changes in aircraft launch time and speed. Selecting the solution least sensitive to changes in parameters allows the network to perform optimally over a wide range of input values.

Due to the infrequent use and highly variable demand for coverage of the Type II areas, this research recommends that Type II areas be excluded from consideration in the permanent network. Type II areas also change frequently, which would require the network to be changed each time a new Type II area was introduced or an old area was removed. To provide network stability, this research recommends covering Type II areas with deployed assets as needed.

Limitations

This research is limited by the accuracy of the data and objectives provided by the ACC Department of Homeland Security and the First Air Force Air Operations Center. Different objectives might change the suitability of the location modeling techniques to this problem. Also, desired response times, aircraft launch times, and aircraft speeds falling out of the range used in this research would produce different results. This research assumes unlimited coverage capacity at the alert sites. Limiting the amount of areas that can be covered by any given site might affect the optimal solutions generated. This research does not consider the existing infrastructure or the cost of obtaining the necessary infrastructure in its location analysis. If costs were incorporated, the optimal

solution could change as well. Also, explosive quantity distance requirements are not considered in the site requirements of this research.

The variability of aircraft launch times and aircraft speeds are not considered in this research. Different aircraft types have different operating characteristics and intercept capabilities. Additionally, the models do not take into account the stochastic or random nature of demand and the fact that aircraft at the alert facility in the optimal solution might not be available when called upon to intercept. The models also fail to recognize the temporal and spatial variation in the actual intercepts. Finally, this research does not consider political weights or objectives in the siting of the optimal network solutions. Each site has an equal probability of selection.

Future Research

Further research needs to be conducted to examine the infrastructure costs associated with the different candidate alert sites. Also, the explosive quantity distance requirement should be examined for all sites. Finally, political desirability of one site over another could be accomplished through weighting. When costs and explosive quantity distance requirements are considered, the optimal solutions will most likely change, because some locations have existing infrastructure, which could make them more desirable candidates. All of these characteristics could be incorporated into a multi-objective optimization model or heuristic solution method. Also, the models presented in this research could be run by utilizing the actual probability distributions for launch times and aircraft speeds. This could generate different network solutions given the probabilistic variation.

Finally, since this research does not investigate the probability of successfully intercepting a threat within a particular area of interest, simulation could be used to evaluate the performance of the optimal network(s) generated in this research against different scenarios. For example, terrorist attacks could be simulated in different areas and the overall network performance examined. This could be done by interviewing intelligence analysts to ascertain likely real world scenarios and incorporating them into the simulation model.

Summary of Findings

From the start of this research effort, the main objective was to determine the optimal strip alert network configuration given the objectives of the ACC Department of Homeland Security, NORAD, and First Air Force through the avenue of location modeling. The model generated in this research effort delivered the optimum network configuration (s) given a wide range of parameters, but also demonstrated that any optimum solution is critically dependent on the desired objectives of the model as well as the values of the input parameters.

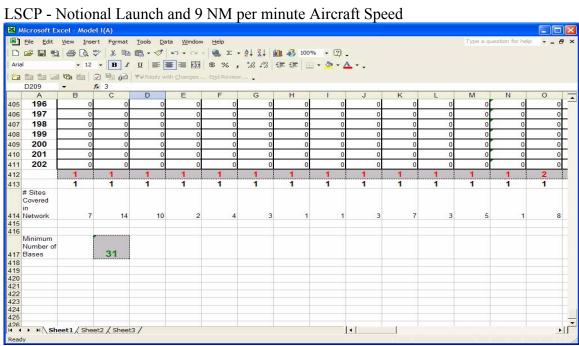
Appendix A. C++ Code Geographic Distance Calculator

```
#include <stdio.h>
#include <stdlib.h>
#include <conio.h>
#include <time.h>
#include <math.h>
#include <iostream.h>
#define rad .017453293
#define n 205
#define supply locations 202
#define demand locations 70
int i,j;
float lat1,long1,lat2,long2,a,b;
float huge d[supply locations+1][n+1],dlat[n+1],slat[n+1],dlong[n+1],slong[n+1];
float difflong, difflat;
void input data(void);
void output data(void);
void main()
cout << "computing nautical mile distances....";
input data();
//Haversine Distance Calculations using Latt and Long//
//Remember that West Long and South Lat are Negative Values//
for (i=1;i<=supply locations;i++)
lat2=slat[i];
long2=slong[i];
lat2=lat2*rad; //Convert Decimal Degrees to Radians for Trig Calculations//
long2=long2*rad;
for(j=1;j \le demand locations;j++)
 lat1=dlat[i];
 long1=dlong[j];
 lat1=lat1*rad; //Convert Decimal Degrees to Radians for Trig Calculations//
 long1=long1*rad;
 difflat=(lat2-lat1); //Calculate Distance//
 difflong=(long2-long1);
```

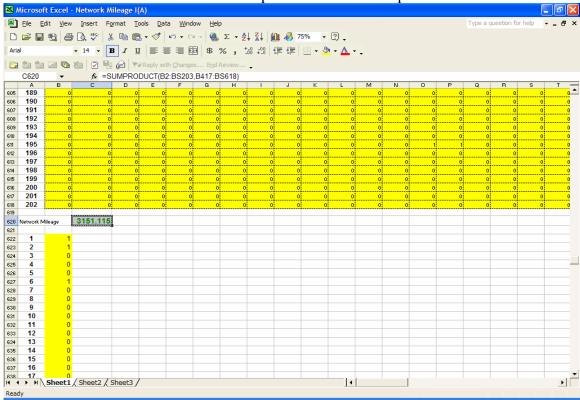
```
a=(sin(difflat/2)*sin(difflat/2))+(cos(lat1)*cos(lat2)*sin(difflong/2)*sin(difflong/2));
 b=2*atan2(sqrt(a), sqrt(1-a));
 d[i][j]=b*3437.67; //nautical miles...3956 US standard miles//
output data();
cout<<".....done!";
void input data()
FILE *fin1,*fin2,*fin3,*fin4;
fin1=fopen("slat.txt","r");
fin2=fopen("slong.txt","r");
fin3=fopen("dlat.txt","r");
fin4=fopen("dlong.txt","r");
for (i=1;i<=supply locations;i++)
 fscanf(fin1,"%f",&slat[i]);
 fscanf(fin2,"%f",&slong[i]);
for (i=1;i \le demand locations;i++)
 fscanf(fin3,"%f",&dlat[i]);
 fscanf(fin4,"%f",&dlong[i]);
fclose(fin1);
fclose(fin2);
fclose(fin3);
fclose(fin4);
void output data()
FILE *fin1;
fin1=fopen("ebdist.xls","w");
for(i=1;i<=supply locations;i++)
for(j=1;j \le demand locations;j++)
 fprintf(fin1,"\t%.4f",d[i][j]);
fprintf(fin1,"\n");
```

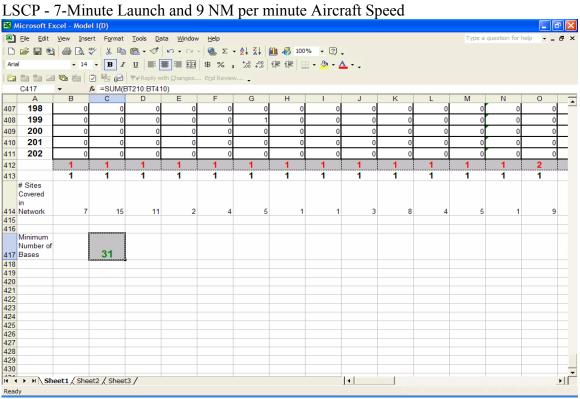
```
fclose(fin1);
}
```

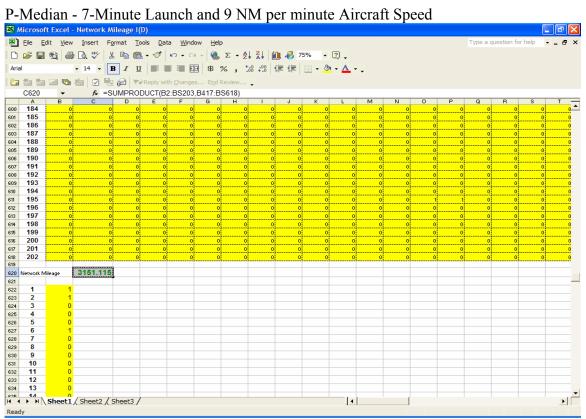
Appendix B. Snapshots of Model Set I Solver™ Run Results in Microsoft Excel®

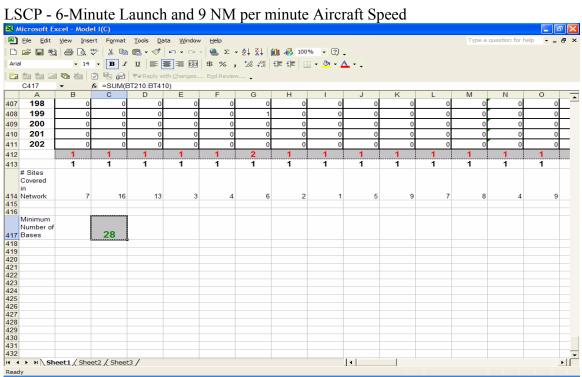


P-Median - Notional Launch and 9 NM per minute Aircraft Speed

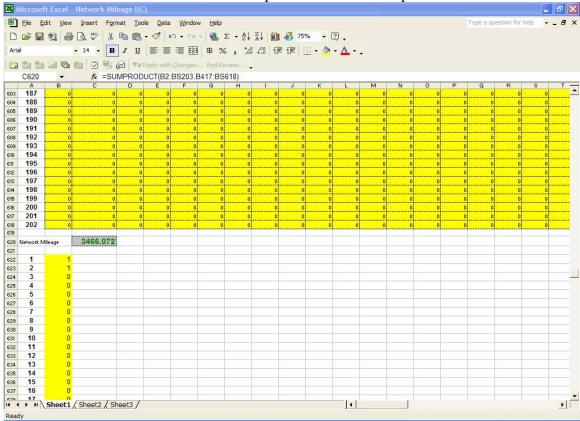


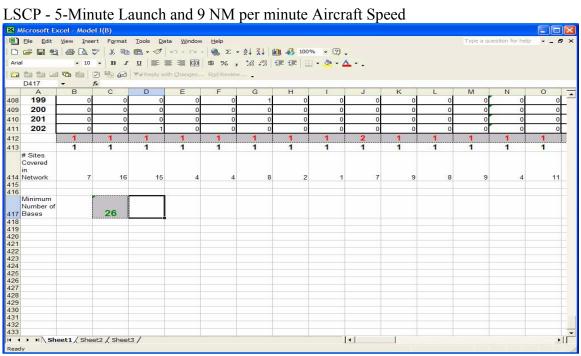




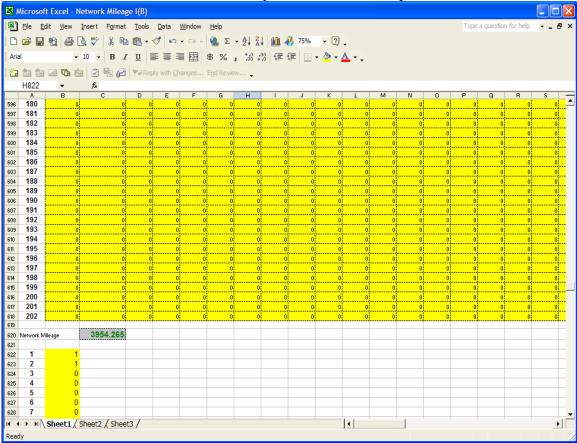


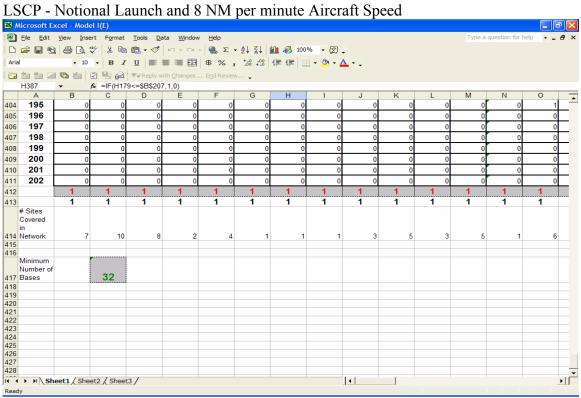
P-Median - 6-Minute Launch and 9 NM per minute Aircraft Speed



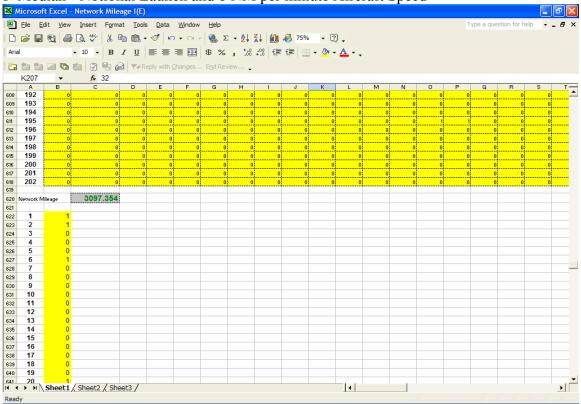


P-Median - 5-Minute Launch and 9 NM per minute Aircraft Speed



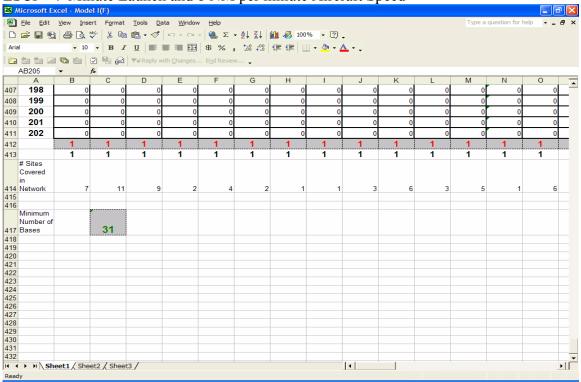


P-Median - Notional Launch and 8 NM per minute Aircraft Speed

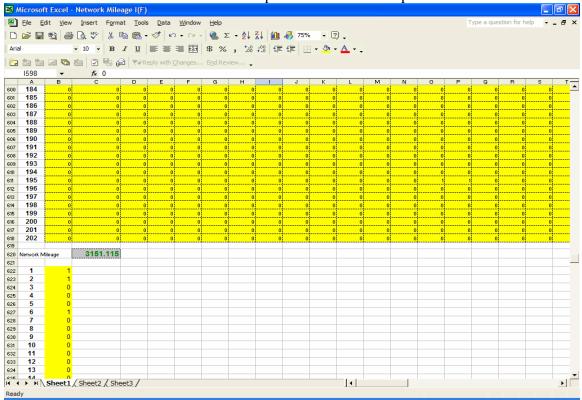


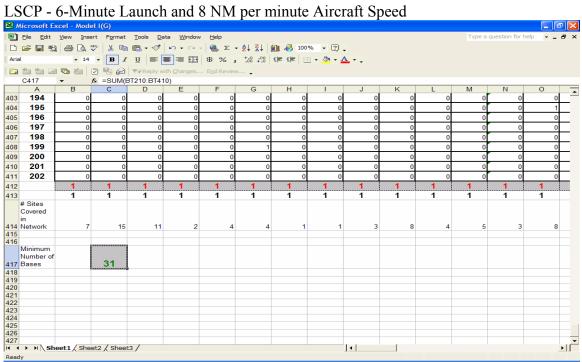
LSCP - 7-Minute Launch and 8 NM per minute Aircraft Speed

Microsoft Excel - Model I(F)

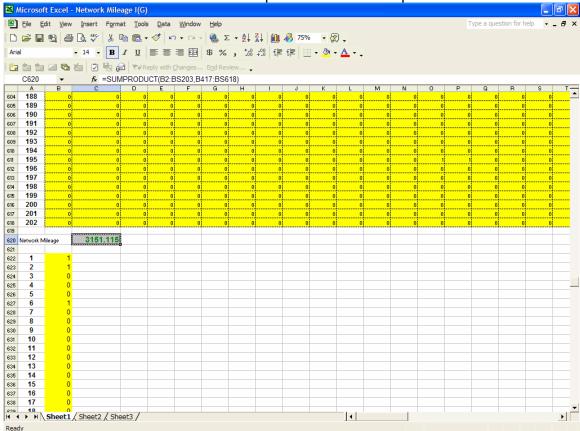


P-Median - 7-Minute Launch and 8 NM per minute Aircraft Speed

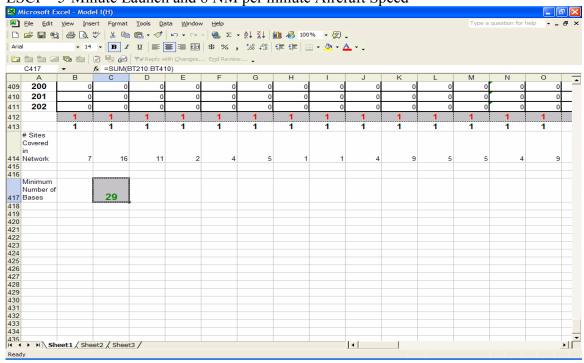




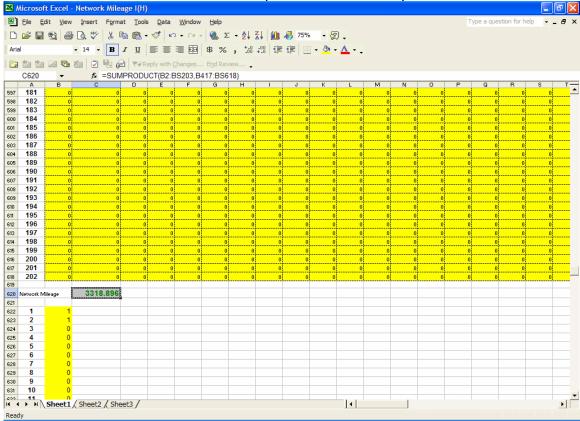
P-Median - 6-Minute Launch and 8 NM per minute Aircraft Speed



LSCP - 5-Minute Launch and 8 NM per minute Aircraft Speed

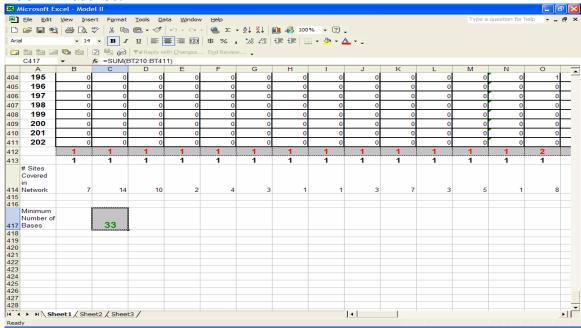


P-Median - 5-Minute Launch and 8 NM per minute Aircraft Speed

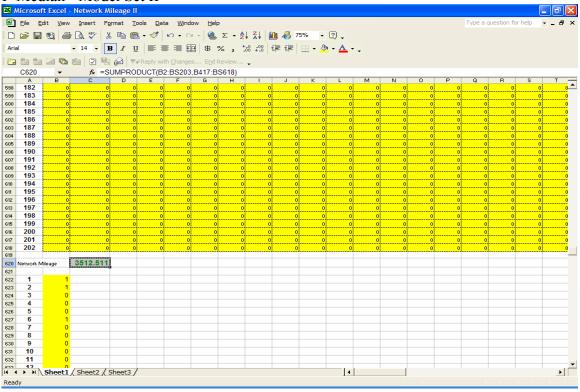


Appendix C. Snapshots of Model Set II Solver™ Run Results in Microsoft Excel®

LSCP - Model Set II

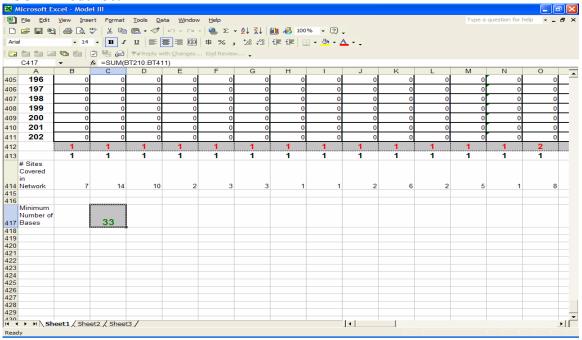


P-Median - Model Set II

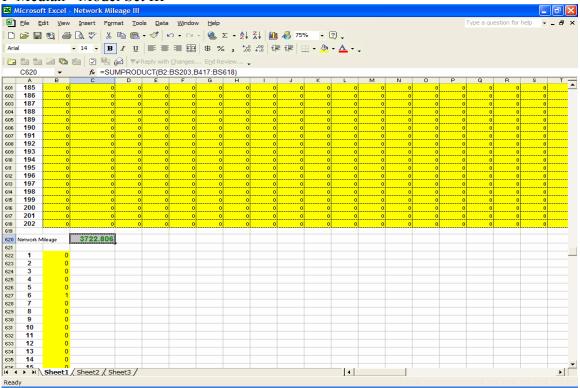


Appendix D. Snapshots of Model Set III Solver™ Run Results in Microsoft Excel®

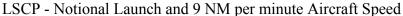
LSCP - Model Set III

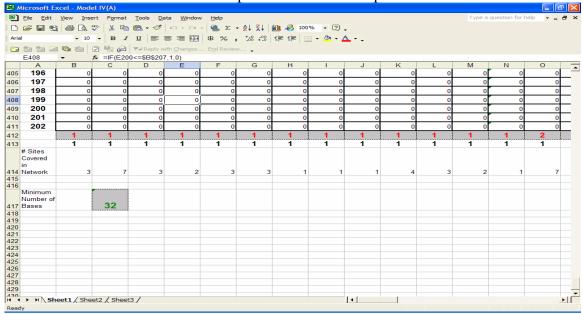


P-Median - Model Set III

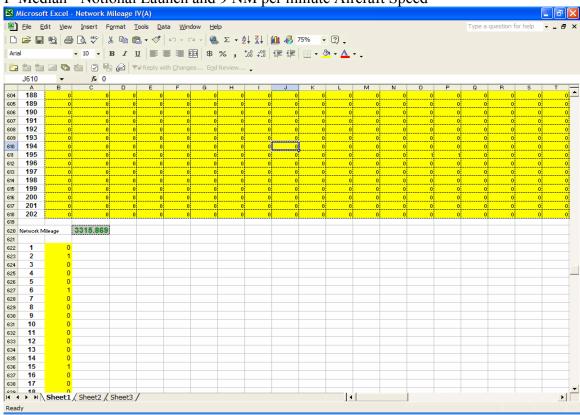


Appendix E. Snapshots of Model Set IV Solver™ Run Results in Microsoft Excel®

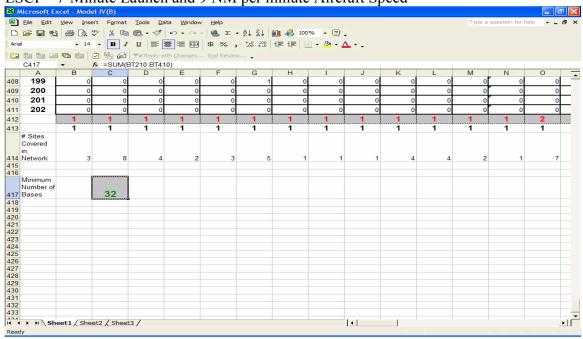




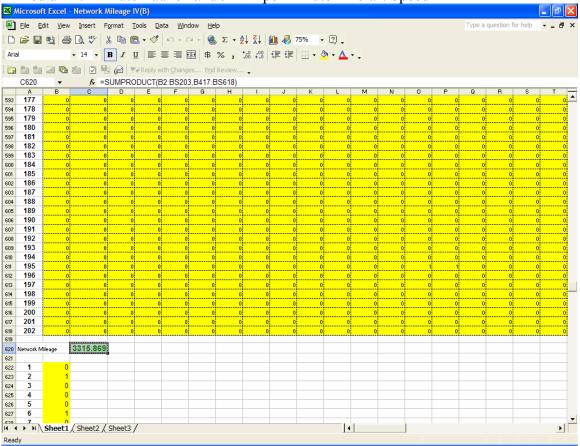
P-Median - Notional Launch and 9 NM per minute Aircraft Speed



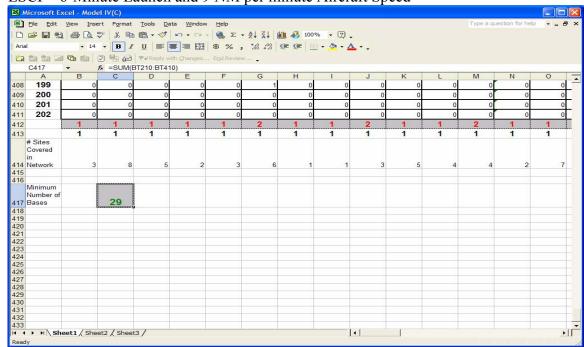
LSCP - 7-Minute Launch and 9 NM per minute Aircraft Speed



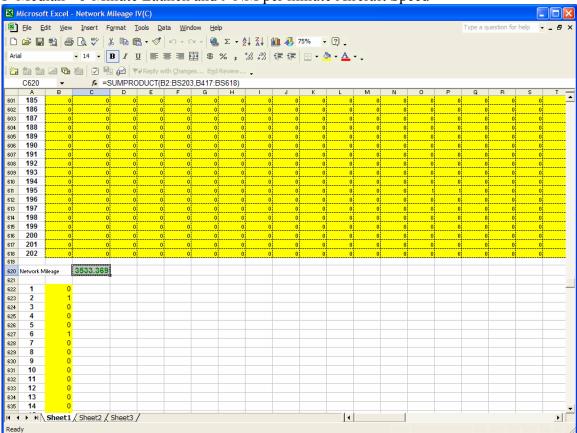
P-Median - 7-Minute Launch and 9 NM per minute Aircraft Speed



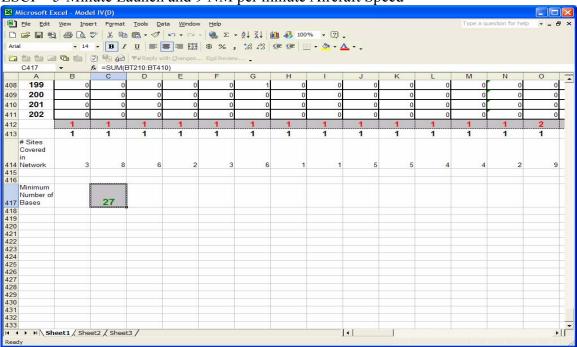
LSCP - 6-Minute Launch and 9 NM per minute Aircraft Speed



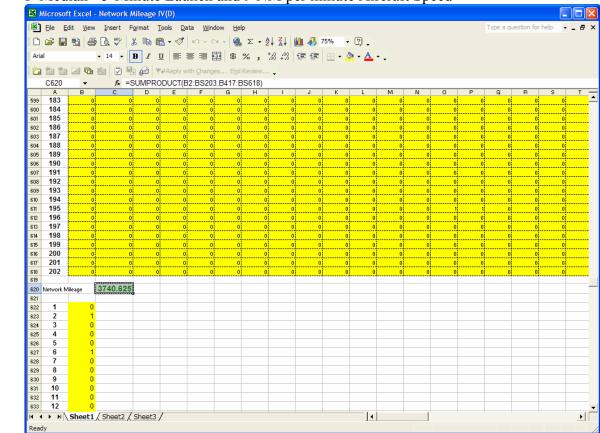
P-Median - 6-Minute Launch and 9 NM per minute Aircraft Speed



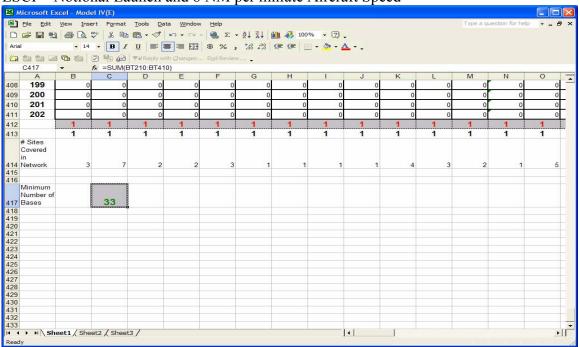
LSCP - 5-Minute Launch and 9 NM per minute Aircraft Speed



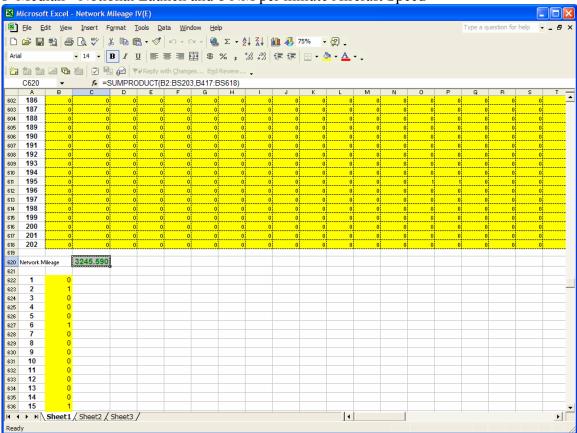
P-Median - 5-Minute Launch and 9 NM per minute Aircraft Speed



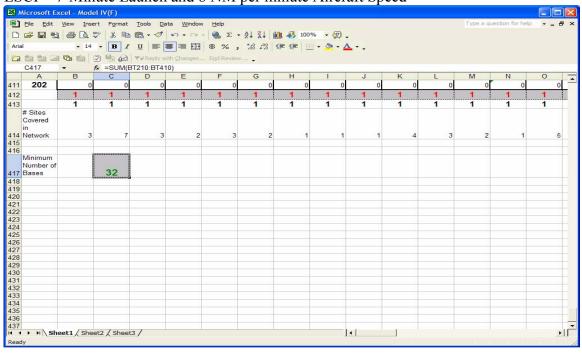
LSCP - Notional Launch and 8 NM per minute Aircraft Speed



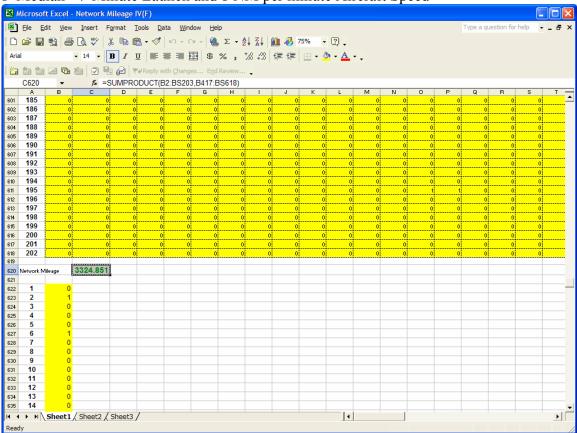
P-Median - Notional Launch and 8 NM per minute Aircraft Speed



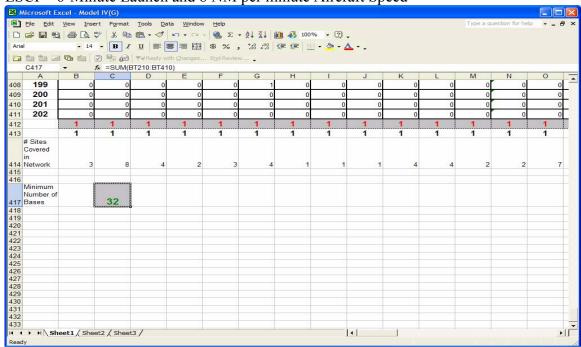
LSCP - 7-Minute Launch and 8 NM per minute Aircraft Speed



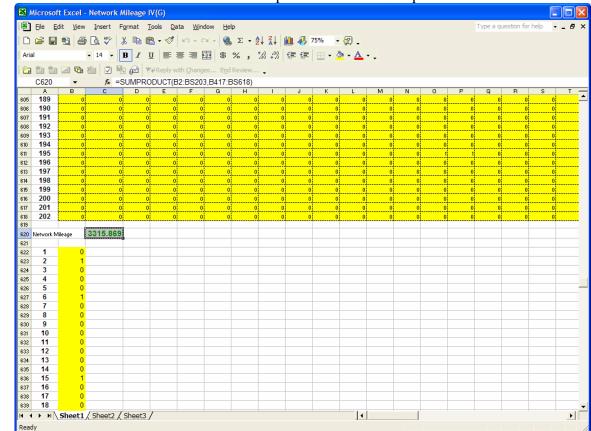
P-Median - 7-Minute Launch and 8 NM per minute Aircraft Speed



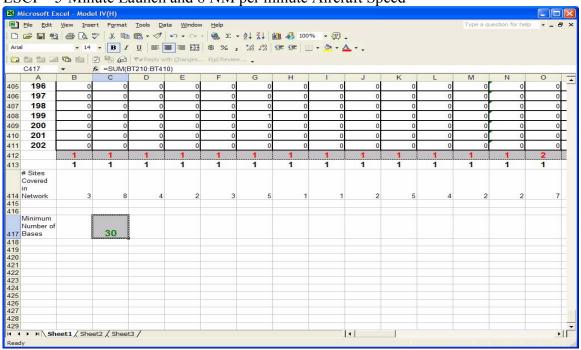
LSCP - 6-Minute Launch and 8 NM per minute Aircraft Speed



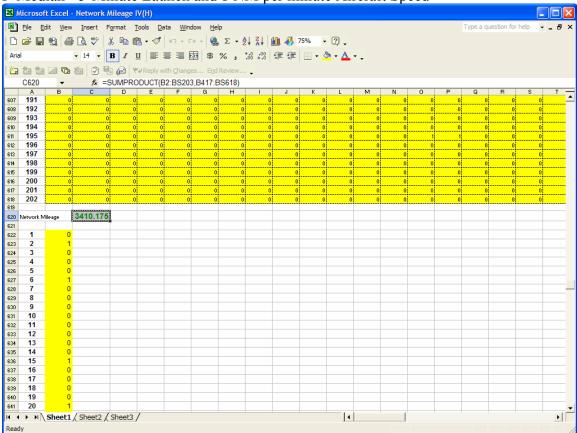
P-Median - 6-Minute Launch and 8 NM per minute Aircraft Speed



LSCP - 5-Minute Launch and 8 NM per minute Aircraft Speed



P-Median - 5-Minute Launch and 8 NM per minute Aircraft Speed



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Vita

Captain Jon A. Eberlan graduated from Jasper High School, Jasper, Texas, in May 1987. He enlisted in 1990 as an Electronic Warfare Systems Specialist, attaining the rank of staff sergeant during a 7-year enlisted career, with assignments at Shaw AFB, South Carolina; Kunsan AB, Korea; and Tinker AFB, Oklahoma. He earned honors as the 20th Component Repair Squadron (CRS) Airman of the Year and 20th CRS Maintenance Professional of the Year of 1993. He was a Distinguished Graduate and the Leadership Award winner for his Airman Leadership School class.

Captain Eberlan graduated with a Bachelor of Science degree in Business

Management with honors from Park College in 1997. He was commissioned through

Officer Training School in 1998 and was a Distinguished Graduate. After

commissioning, Captain Eberlan attended the Aircraft Maintenance Officer's Course and
earned honors as a Distinguished Graduate and was voted the Top Graduate.

In December 1998, he was assigned to Davis-Monthan AFB, Arizona. He was decorated for his actions in Operation ALLIED FORCE, and was the 1999 CGO of the Year for the 42d Airborne Command and Control Squadron. He was recognized as the 355th Equipment Maintenance Squadron CGO of the Year, 2000. Finally, he earned honors as the 355th CRS CGO of the Year, 355th Logistics Group CGO of the Year, 355th Wing CGO of the Year, and 12 AF CGO of the Year for 2001. In August 2002, he entered the Graduate School of Engineering and Management, Air Force Institute of Technology. Upon graduation, he will be assigned to Langley AFB, Virginia. Captain Eberlan is married and has three children.

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